Analysis of anisotropic and isotropic models for estimation of solar radiation on the inclined surface in New Delhi location

Saibal Manna* and Ashok Kumar Akella
Department of Electrical Engineering, NIT Jamshedpur, Jharkhand, 831014, India

*E-mail: mannasail1994@gmail.com

Manuscript Received online 6/01/2020, Accepted 8/21/2020

This content aims to compare several experimental models utilized for the estimation of sunlight based radiation on inclined plane. So, three anisotropic and equal quantities of isotropic models were engaged regularly in New Delhi and their outcomes were analyzed for choice of correct and fitting model for this territory. Three isotropic models to be specific BA (2002), LJ (1960) and KO (1986) and three anisotropic models to be specific HDKR (2006), HD (1980) and RE (1990) model were explored. Here tilt angle was adjusted at 28.58° N (New Delhi latitude). The result of six models had been compared with ground measured data. For this five statistical tests are used for comparison. HD evaluated the most noteworthy measure of occurrence sun oriented radiation in the entire year while BA set up the least among entire models. LJ and KO model showed similar outcomes. The outcomes of statistical analysis gave that HDKR provided smaller MAPE (1.02%), MBE (0.129 kWh/m²-day), RMSE (0.605 kWh/m²-day), RMSRE (0.393 kWh/m²-day) and RRMSE (1.858%) among all six models. Ultimately, HDKR model was favored for estimation of sunlight based radiation occurrence on an inclined plane with least errors. MATLAB has been used for implementation.

Keywords: Anisotropic model, Isotropic model, Inclined plane, Solar radiation, Statistical test

Introduction

Sun oriented radiation information is the best wellspring of data for evaluating normal occurrence radiation essential for appropriate structure and the appraisal of sunlight based energy transformation frameworks. There are a few types of solar radiation data, which could be utilized for an assortment of purposes in the structure and improvement of sun oriented energy frameworks. Hourly radiation may be estimated from day by day information. A few models are created to evaluate the Mg utilizing different climate specifications, for example, daylight length, temperature, moisture and speed of wind. The author utilized the metrological information (1994-2005) of china to evaluate regular global radiation from various parameter like air temperature etc. The author newly projected a straightforward model for evaluation of Mg on flat plane for sixty eight states of Turkey. A model is presented with high aerosol to determine the monthly average hourly global radiation ($I_g$) by utilizing satellite data. It is somewhat crucial to determine direct and indirect segments of total radiation fact on flat plane. When these segments are solved, it can be converted over tilted plane and thus PV module and other sun based devices can be evaluated. El-Sebaii introduced interaction for evaluating diffuse radiation by interacting ($M_{gim}$), ($M_{dm}$) and ($L/L_{max}$) in Egypt. A new method is introduced
by the author which might be utilized for evaluating $M_b$ based on the elevation angle\(^8\). ANN-based satellite data were utilized to evaluate beam and indirect radiation in various town of Turkey\(^9\). Radiation occurrence on inclined plane comprises three parts: ground reflected, beam and diffuse radiation. Solar radiation directly gains on earth’s plane is termed as direct radiation. Radiation attains on earth’s surface after having been dispersed by particle in the earth’s atmosphere is known as diffuse radiation. Energy of diffuse radiation is uniform over the sky is called isotropic model. While the anisotropic models assume that the energy of diffuse radiation is nonuniform over the sky.

The fundamental destination of this study are:

1. Evaluate the $M_g$, $M_b$, $M_d$ on flat plane utilizing several experimental models in New Delhi location.
2. Determine $M_T$ occurrence on inclined plane at tilt angle $28.58°$ N using six chosen experimental models.
3. Analyze every model with estimated and measured data using five statistical test formula.

### Nomenclature

- $M_g$: Monthly average daily global radiation on a horizontal surface
- $C$: Solar constant $= 1.367 \text{ kW/m}^2$
- $P$: Day of the year
- $M_o$: Monthly average daily extra-terrestrial radiation fall on a horizontal surface
- $a_1$, $b_1$: Angstrom constants (New Delhi $a_1 = 0.26$, $b_1 = 0.05$)
- $L$: Monthly average daily hours of bright sunshine (hours)
- $L_{max}$: Monthly average of the maximum possible daily hours (day length) of bright sunshine
- $M_{d}$: Monthly average daily defused radiation
- $C_I$: Monthly average cleanness index
- $M_T$: Total incident solar radiation on tilted surface
- $M_{T,b}$: Tilted surface beam radiation
- $M_{T,d}$: Tilted surface diffuse radiation
- $M_{T,r}$: Tilted surface ground reflected radiation
- $M_{b}$: Monthly average daily beam radiation on horizontal surface
- $T_b$: View factor for beam radiation
- $M_{gm}$: Metrological ground measured global solar radiation at horizontal surfaces
- $M_{gmt}$: Metrological ground measured tilted global solar radiation ($\text{kWh/m}^2$-$\text{day}$)
- HD: Hay and Davies Model
- BA: Badescu Model
- LJ: Liu and Jordan Model
- KO: Koronakis Model
- RE: Reindl et al. Model
- HDKR: Hay and Davies, Klucher Model
- D: Anisotropy index

### Material and method

#### About New Delhi location

The latitude, longitude and altitude of New Delhi is $28.61°$N, $77.21°$ E and 216 m. The atmosphere of New Delhi is sweltering summer and dry winter. The normal climate is $30°C$ and $40°C$ during May. The overall rainfall of the city is below 1200mm.

#### Solar radiation on horizontal Surface

The $M_o$ is expressed by given equation

$$ M_o = \frac{24}{\pi} C \left(1 + 0.033 \cos \frac{360P}{365} \right) \times \left( \frac{a_1 \sin \varphi \sin \lambda + b_1 \cos \varphi \cos \lambda \sin \mu_r}{180} \right) $$

(1)

The declination angle ($\lambda$) is calculated from the below equation\(^{10}\)

$$ \lambda = 23.45 \sin \frac{360}{365} (284 + P) $$

(2)
Where P is total no of day as appear in Table 1.
The sunshine hour angle ($\mu_s$) is determined by 
\[
L_{\text{max}} = \left(\frac{2}{15}\right)\mu_s, \mu_s = \cos^{-1}\left(-\tan\lambda \tan\varphi\right) (3)
\]

### Table 1. Total number of days corresponding to month

<table>
<thead>
<tr>
<th>Months</th>
<th>Days (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>15</td>
</tr>
<tr>
<td>Feb.</td>
<td>46</td>
</tr>
<tr>
<td>Mar.</td>
<td>75</td>
</tr>
<tr>
<td>Apr.</td>
<td>106</td>
</tr>
<tr>
<td>May.</td>
<td>136</td>
</tr>
<tr>
<td>Jun.</td>
<td>166</td>
</tr>
<tr>
<td>Jul.</td>
<td>196</td>
</tr>
<tr>
<td>Aug.</td>
<td>227</td>
</tr>
<tr>
<td>Sept.</td>
<td>258</td>
</tr>
<tr>
<td>Oct.</td>
<td>288</td>
</tr>
<tr>
<td>Nov.</td>
<td>319</td>
</tr>
<tr>
<td>Dec.</td>
<td>349</td>
</tr>
</tbody>
</table>

The $M_g$ is given by
\[
\frac{M_g}{M_0} = a_1 + b_1 \left(\frac{L}{L_{\text{max}}^2}\right) (4)
\]

The $L_{\text{max}}$ can be calculated:
\[
L_{\text{max}} = \left(\frac{2}{15}\right)\mu_s (5)
\]

The $M_b$ is determined by:
\[
\frac{M_d}{M_g} = 1.411 - 1.696 \left(\frac{M_g}{M_0}\right) (6)
\]

Global radiation is obtained by adding diffuse and beam radiation. So, Beam radiation is expressed as
\[
M_b = M_g - M_d (7)
\]

**Solar radiation on inclined plane**
The $M_T$ is given as:
\[
M_T = M_{T,b} + M_{T,r} + M_{T,d} (8)
\]

**Beam radiation ($M_{T,b}$)**
The radiation on tilted surface is given by:
\[
M_T = M_b T_b (9)
\]

$M_b$ is calculated by equation 7. The value of $R_b$ is calculated by:
\[
T_b = \frac{\sin \lambda \sin (\varphi - \alpha) + \cos \lambda \cos (\varphi - \alpha)}{\sin \lambda \sin \varphi + \cos \lambda \sin \varphi \cos \mu} (10)
\]

Where $\mu$ is the hour angle and $\alpha$ is the inclination of tilted surface (degree).

**Reflected radiation ($M_{T,r}$)**
The radiation reflected from earth’s surface is called reflected radiation. It ($M_{T,r}$) can be obtained from below equation:
\[
M_{T,r} = M_k \eta \left(\frac{1 - \cos \alpha}{2}\right) (11)
\]

Where $\eta$ is the constant (ground reflectance). Here consider $\eta = 0.2$ which is most commonly used for hot location^{14}.

**Diffused radiation ($M_{T,d}$)**

After scattering the radiation gained at the earth’s plane from entire parts of the sky in the atmosphere is called diffuse radiation. Condition of cloudiness and atmospheric clearness is the function of this radiation which are extremely unpredictable. Horizon brightening, isotropic and circumsolar are the three components of this radiation.

**Anisotropic and Isotropic models**
The models are categorized as anisotropic and isotropic sky models. For this, six models were picked, and their outcomes were analyzed for choice of correct and fitting model for this territory. Three isotropic models to be specific BA (2002), LJ (1960)^{15} and KO (1986)^{16} and three anisotropic models to be specific HDKR (2006), HD (Hay 1980)^{17} and RE (1990)^{18} model were explored.
A short portrayal of the anisotropic and isotropic models chose for correlation of evaluated results is given underneath:

**LJ model**

Here horizontal brightening and circumsolar were considered as zero. The complete expression for calculating $M_T$ is given below:

$$M_T = M_b T_b + M_g \eta \left( \frac{1 - \cos \alpha}{2} \right) + M_d \left( \frac{1 + \cos \alpha}{2} \right)$$

**KO model**

For this model the $M_T$ will be

$$M_T = M_b T_b + M_g \eta \left( \frac{1 - \cos \alpha}{2} \right) + M_d \left( \frac{2 + \cos \alpha}{3} \right)$$

**BA model**

The $M_T$ for this model is shown below:

$$M_T = M_b T_b + M_g \eta \left( \frac{1 - \cos \alpha}{2} \right) + M_d \left( \frac{3 + \cos 2 \alpha}{4} \right)$$

**HD model**

The $M_T$ on an inclined plane is given as follows.

$$M_T = (M_b + M_d D) T_b + M_g \eta \left( \frac{1 - \cos \alpha}{2} \right) +$$

$$M_d \left( \frac{1 + \cos \alpha}{2} \right) (1 - D) + AT_b$$

Where $D$ is anisotropy index, is expressed as

$$D = \frac{M_b}{M_o}$$

**RE model**

Their proposed model is given underneath:

$$M_T = (M_b + M_d D) T_b + M_g \eta \left( \frac{1 - \cos \alpha}{2} \right) +$$

$$M_d \left( (1 - D) \left( \frac{1 + \cos \alpha}{2} \right) \left[ 1 + \frac{M_b}{M_b} \sin^2 \left( \frac{\alpha}{2} \right) + DT_b \right] \right)$$

**HDKR model**

This model is partner with HD, Klucher and RE models. The model is given underneath:

$$M_T = (M_b + M_d D) T_b + M_g \eta \left( \frac{1 - \cos \alpha}{2} \right) +$$

$$M_d \left( (1 - D) \left( \frac{1 + \cos \alpha}{2} \right) \left[ 1 + \frac{M_b}{M_b} \sin^2 \left( \frac{\alpha}{2} \right) + DT_b \right] \right)$$

**Methods of models evaluation**

Here Indian meteorological department data considered as measured data. Now, estimated global radiation on a flat and inclined plane is compared with measured data. For this, five statistical tests are used for comparison.

- Mean Absolute Percentage Error (MAPE)
- Mean Bias Error (MBE)
- Root Mean Square Error (RMSE)
- Root Mean Squared Relative Error (RMSRE)
- Relative Root Mean Squared Error (RRMSE)

These tests assess the exactness of the connections portrayed previously.

**MAPE**

This error is a symbol of precision which generally gives exactness as a percentage of the data. It might be communicated as:

$$MAPE = \frac{1}{m} \sum_{j=1}^{m} \left( \frac{V - V_p}{V} \right) \times 100$$

Where $V$ is measured value, $V_p$ is estimated Value, and $m$ is the total number of Perceptions.

**MBE**

This error is given by:

$$MBE = \frac{1}{m} \sum_{j=1}^{m} (V_p - V_j)$$
Where $V_j$ is $j^{th}$ measured Value, $V_{pj}$ is $j^{th}$ estimated Value

**RMSE**

This Error might be computed from the below expression:

$$RMSE = \sqrt{\frac{1}{m} \sum_{j=1}^{m} (V_{pj} - V_j)^2}$$

Where $V_j$ is $j^{th}$ measured Value, $V_{pj}$ is $j^{th}$ estimated Value

**RMSRE**

This error is determined by dividing RMSE with mean value of measured data.

$$RMSRE = \sqrt{\frac{1}{m} \sum_{j=1}^{m} \left( \frac{V - V_{p}}{V} \right)^2} \times 100$$

**Result and Discussions**

*Solar radiation on horizontal plane*

After calculation, the value of $\lambda$, $\mu_s$, $M_g$ and $M_o$ is given in Table 2 and the variation of $M_o$, $M_g$, $M_d$, $M_{gm}$ and $M_{gmt}$ on horizontal surface are shown in Fig. 1. $M_o$ is seen to be highest in June 11.395 and least in December 5.719 kWh/m²-day. $M_g$ is evaluated with the help regression constant for New Delhi ($a_1 = 0.25$ and $b_1 = 0.50$).  

*Sky condition of New Delhi*

The clearness index ($C_I$) is the parameter that demonstrates the straightforwardness of the environment and showed by division of extraterrestrial radiation that arrives at the earth’s surface as global sunlight based radiation. $C_I$ is characterized as $C_I = M_g/M_o$. $C_I$ is determined from the estimated value of $M_o$ and $M_g$. $C_I$, $M_d/M_o$ and $M_d/M_g$ for New Delhi appear in Fig. 2.
Fig. 3. Analysis of various models with $M_T$ at New Delhi.

Variety of estimated solar radiation on inclined plane with various models

The observation declared that LJ, KO and BA model exhibited around same outcomes. RE and HDKR Model execute large value than LJ, BA and KO model as shown in Fig. 3. HD model showed the highest values among all models. BA exhibited the lowest value than all models. It was established from inspection that all models forecast higher incident solar energy irradiation on inclined plane ($M_T$) than on horizontal plane ($M_g$) because of the slope optimization. The value of $M_T$ for six models are given in Table 3.
Table 3. $M_T$ (kWh/m$^2$-day) by six models and measured data at New Delhi.

<table>
<thead>
<tr>
<th>Month</th>
<th>$M_{gmt}$</th>
<th>LJ</th>
<th>KO</th>
<th>BA</th>
<th>HD</th>
<th>RE</th>
<th>HDKR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>4.57</td>
<td>4.78</td>
<td>4.80</td>
<td>4.72</td>
<td>7.35</td>
<td>5.72</td>
<td>5.06</td>
</tr>
<tr>
<td>Feb.</td>
<td>5.77</td>
<td>5.13</td>
<td>5.15</td>
<td>5.05</td>
<td>7.62</td>
<td>6.10</td>
<td>5.38</td>
</tr>
<tr>
<td>Mar.</td>
<td>6.7</td>
<td>5.46</td>
<td>5.49</td>
<td>5.36</td>
<td>7.79</td>
<td>6.42</td>
<td>5.67</td>
</tr>
<tr>
<td>Apr.</td>
<td>6.75</td>
<td>5.62</td>
<td>5.66</td>
<td>5.50</td>
<td>7.70</td>
<td>6.51</td>
<td>5.77</td>
</tr>
<tr>
<td>May.</td>
<td>6.46</td>
<td>5.58</td>
<td>5.63</td>
<td>5.46</td>
<td>7.42</td>
<td>6.39</td>
<td>5.68</td>
</tr>
<tr>
<td>Jun.</td>
<td>5.70</td>
<td>5.51</td>
<td>5.56</td>
<td>5.39</td>
<td>7.22</td>
<td>6.27</td>
<td>5.59</td>
</tr>
<tr>
<td>Jul.</td>
<td>5.01</td>
<td>5.52</td>
<td>5.57</td>
<td>5.40</td>
<td>7.28</td>
<td>6.30</td>
<td>5.61</td>
</tr>
<tr>
<td>Aug.</td>
<td>4.99</td>
<td>5.56</td>
<td>5.61</td>
<td>5.45</td>
<td>7.53</td>
<td>6.42</td>
<td>5.69</td>
</tr>
<tr>
<td>Sept.</td>
<td>5.58</td>
<td>5.48</td>
<td>5.52</td>
<td>5.38</td>
<td>7.70</td>
<td>6.42</td>
<td>5.67</td>
</tr>
<tr>
<td>Oct.</td>
<td>5.94</td>
<td>5.20</td>
<td>5.23</td>
<td>5.12</td>
<td>7.63</td>
<td>6.17</td>
<td>5.44</td>
</tr>
<tr>
<td>Nov.</td>
<td>5.23</td>
<td>4.85</td>
<td>4.87</td>
<td>4.79</td>
<td>7.38</td>
<td>5.80</td>
<td>5.12</td>
</tr>
<tr>
<td>Dec.</td>
<td>4.47</td>
<td>4.66</td>
<td>4.68</td>
<td>4.61</td>
<td>7.24</td>
<td>5.60</td>
<td>4.95</td>
</tr>
</tbody>
</table>

Statistical Analysis of models

The outcomes of this analysis are shown in Figs. 4–6. It tends to be seen from Fig. 4. MAPE for BA and HD models are 6.18% and -35.87% respectively while for different models: LJ 4.53%, KO 3.91%, RE -11.82% and HDKR Model 1.02%. MBE is less for HDKR and KO model. From fig 5, value of these two models are 0.1288 and 0.2838 kWh/m$^2$-day respectively. LJ and BA showed similar value with 0.319 and 0.411 kWh/m$^2$-day respectively. HD and RE Model scored 0.189 and 0.580 kWh/m$^2$-day MBE error. RMSE gives data on short term execution of the models. As appeared in Fig. 6, HD model provided the largest value 1.994 kWh/m$^2$-day of RMSE whereas HDKR creating a least RMSE 0.605 kWh/m$^2$-day which is closed to KO 0.658 kWh/m$^2$-day. Other models LJ, RE and BA Model values are 0.674, 0.815 and 0.723 kWh/m$^2$-day. From Fig.7, HDKR gives the lowest RMSRE error among the models and value is 0.104 kWh/m$^2$-day. LJ and KO model gives same values (0.109 and 0.108 kWh/m$^2$-day respectively). HD model gives the highest RMSRE error (0.393 kWh/m$^2$-day). From Fig.8, HDKR provides the least RRMSE error among the models and value is 1.858%. HD model scored the highest RRMSE error.
Table 4 gives the Statistical Evaluation of six different models.

- **Fig. 4.** MAPE for Six models
- **Fig. 5.** MBE for Six models
- **Fig. 6.** RMSE for Six models
- **Fig. 7.** RMSRE for Six models

### Table 4. Statistical Evaluation of six models

<table>
<thead>
<tr>
<th>Model</th>
<th>MAPE</th>
<th>MBE</th>
<th>RMSE</th>
<th>RMSRE</th>
<th>RRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LJ</td>
<td>4.53</td>
<td>-0.319</td>
<td>0.674</td>
<td>0.109</td>
<td>1.954</td>
</tr>
<tr>
<td>KO</td>
<td>3.91</td>
<td>-0.284</td>
<td>0.658</td>
<td>0.108</td>
<td>1.929</td>
</tr>
<tr>
<td>BA</td>
<td>6.18</td>
<td>-0.411</td>
<td>0.723</td>
<td>0.116</td>
<td>2.072</td>
</tr>
<tr>
<td>HD</td>
<td>-35.87</td>
<td>0.189</td>
<td>1.994</td>
<td>0.393</td>
<td>7.020</td>
</tr>
<tr>
<td>RE</td>
<td>-11.82</td>
<td>0.580</td>
<td>0.815</td>
<td>0.165</td>
<td>2.947</td>
</tr>
<tr>
<td>HDKR</td>
<td>1.02</td>
<td>-0.129</td>
<td>0.605</td>
<td>0.104</td>
<td>1.858</td>
</tr>
</tbody>
</table>
Conclusions

The following outcomes are obtained from the analysis of six distinct models at an incline angle of 28.58° N (New Delhi latitude).

1. $M_o$, $M_g$ and $M_d$ were calculated to be 8.86, 4.94 and 1.8 kWh/m²-day on horizontal plane respectively.
2. HD gave the highest and BA showed the least values of $M_T$ among entire models.
3. LJ and KO model showed similar outcomes 5.28 and 5.31 kWh/m²-day.
4. The outcomes of statistical analysis gave that HDKR provided smaller error among all six models.
5. HDKR evaluated radiation more near to the measured value and lowest errors. Subsequently, HDKR can be favored for the evaluation of solar radiation on the inclined plane in New Delhi.
6. These six models can be executed all over the nation where ground measured data is accessible. This can be utilized in solar photovoltaic applications in future.

References