Mathematical modeling for drying of *Solanum tuberosum* under the indirect type solar dryer

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In this study, the solar drying behavior of *Solanum tuberosum* or potato food material has been investigated. A heat and mass transfer-based mathematical model has been developed to predict the temperature and moisture distribution throughout the food sample with its thickness and drying time. An indirect type of solar dryer in which solar radiation is not fallen directly on the food sample has also been fabricated and experimental work has been conducted with a food sample for six hours. The complex partial differential model equations have been solved by the MATLAB 2019a software and represent the three-dimensional surface plot of the distributed temperature and moisture content in the food sample. Global radiation has also been examined along with drying time because global radiation causes the temperature variation inside the solar dryer. Finally, the model result data compared with experimental data and found in good agreement with these two with a 5% error. Therefore, the developed model can be used for some other food materials such as carrot, yam, radish, etc.

Keywords: Solar drying, modeling, global radiation, MATLAB 2019a.

Introduction

Solar energy is an everlasting, renewable source of energy at our earth's surface. Solar drying is the oldest food preservation technique and used for the preservation of vegetables, fruits, grains, and seafood. This drying technique is also helpful for heat-sensitive food material to maintain their texture, color, and taste during the heating period with uniform heating throughout the food material[1,5,17]. Solar drying is a food preservation technology under the action of an indirect type of solar dryer. The solar dryer has low maintenance cost and easy to operate. The solar dryer is artificial drying equipment that can harvest solar radiation and which is used for foodstuffs. It can be operated under the low sunshine i.e. hazy and cloudy conditions also[2,23].

As we know that the experimental work is costly and expensive, so to study the morphology of different types of food samples, moisture content temperature distribution, and other parameters, we developed a mathematical model to predict the behaviour of the foodstuffs[11,13,18,20]. Modeling is also helpful for the optimization of the drying process timely. A modeling study of a food material is based on the transport phenomena in which heat, mass, and momentum transfer have been considered along with some other parameters[3,4,9,24].

Earlier researchers have performed numerous experimental work regarding the air temperature, humidity, mass loss from the food material, solar radiation. They have studied the parameters based on drying time[6,8,10,15].

The present study is dealing with modeling and simulation of the solar-dried *Solanum tuberosum* based on food material's thickness. *Solanum tuberosum* is a rich source of fibre, vitamin C, and vitamin B6; helpful for reducing cholesterol and reducing the risk of a heart attack. The advantage of this study is the solution of model equations has been done by MATLAB software, while earlier researchers have solved the equations by finite difference method. In this article, the analysis of moisture and temperature distribution inside the solar-dried food sample has been done for different thicknesses of the food material. Experimental work has also been done with a fabricated indirect type solar dryer

and evaluated the required parameters to validate the developed mathematical model. This study may helpful for food industries to examine the drying behaviour of food samples computationally even without performing any experiment work.

Materials and methods

Experimental work has been done at the rooftop of the Institute of Engineering and Technology, Lucknow, Uttar Pradesh, India (26°5/N latitude, 80°56/E longitude, 128 m above the sea level) with our fabricated solar dryer set up shown in Fig. 1.

Collection and pre-treatment of the samples:

The potato food sample is easily available in the local market of Lucknow city and has a significant moisture content (avoid dry sample). During the pre-treatment, the samples were washed with running water and peeled it, then; samples were cut in required dimensions such as 35 mm x 30 mm for different thickness i.e. 1, 2, 3, 4, and 5 mm with the help of a sharpened knife.

Apparatus used:

Indirect type of solar dryer, weighing machine (Citizen, 220 CY), IR thermometer (ACETEQ, MT-4), and solar power meter (TENMARS, TM-206).

Fig. 1. (A) Indirect type solar dryer experimental setup, (B) weighing machine, (C) infrared thermometer and (D) solar power meter.

Experimental procedure

After pre-treatment, the initial weight of all five samples has been calculated one by one with the help of a weighing machine. Then all samples were placed on the feed tray in the drying chamber of the solar dryer and the initial temperature of samples was recorded. After one hour, the status of the weight loss and temperature increment in all five food samples were checked. This process was repeated every hour until the food samples have attained equilibrium moisture content conditions.

Mathematical modeling

Simultaneous heat and mass transfer-based mathematical model has been developed to predict the temperature and moisture distribution inside the *Solanum tuberosum* food sample along with the thickness of the food sample and drying time. We have considered a control volume with thickness dx of the sample as shown in Fig. 2 and applied the simultaneous heat and mass balances[28,30].

There are some assumptions related to this study:

(i) The *Solanum tuberosum* food material is considered to be compact.

(ii) The solar dryer is fully insulated.

(iii) In the solar dryer, the air is at a constant temperature.

(iv) One dimensional (x-direction) heating takes place.

(v) Initial temperature and moisture content is uniform.

Now, as per the law of conservation of energy, we get the following equation[12,25];

$$\rho_s C_{p_s} T_{s_t} = K_s T_{s_{xx}} \tag{1}$$

where, $T_{s_{xx}}$ a derivative of food product temperature for change in thickness x and Q_{abs} is absorbed radiant energy
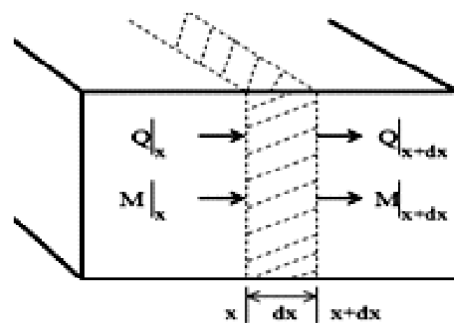

Fig. 2. Control volume of a solar-heated *Solanum tuberosum*.

(Watt) and it can be expressed as;

$$Q_{abs} = \alpha\,(A)q_i \tag{2}$$

and

$$q_i = \frac{q_h}{\cos\theta} \tag{3}$$

Similarly, as per the law of conservation of mass;

$$M_{s_t} = D_{eff}\,M_{s_{xx}} \tag{4}$$

where $M_{s_{xx}}$ a derivative of food product moisture for change in thickness x and D_{eff} is effective diffusivity and it can be written as;

$$D_{eff} = D_0\exp\left(-\frac{E_a m}{Q_{abs}}\right) \tag{5}$$

Eq. (1) and eq. (4) are the partial differential equations with two dependent variables such as drying time 't' and sample thickness 'x'. To solve these equations initial and boundary conditions are given below[4];

Initial conditions:

At time t = 0 and 0 < x < H;

$$T_s = T_{s0} \tag{6}$$

and

$$M_s = M_{s0} \tag{7}$$

Boundary conditions:

At time t > 0 and x = 0;

$$T_{s_x} = 0 \tag{8}$$

and

$$M_{s_x} = 0 \tag{9}$$

At time t > 0 and x = H;

$$T_{s_t} = \frac{1}{dx\,\rho_s C_{ps}}\,[-K_s T_{s_x} + dx\,Q_{abs}] \tag{10}$$

and

$$M_{s_t} = \frac{1}{dx}\,[-D_{eff}\,M_{s_x} - J_{m,s}] \tag{11}$$

Some other parameters like density (ρ_s), specific heat (C_{p_s}), thermal conductivity (K_s), pre-exponential factors (D_0), and mass flux ($J_{m,s}$) are calculated from the literature by using initial temperature and moisture conditions[7,21,22,24].

Numerical solution of the mathematical model

In previous literature, researchers have solved the theoretical model with the help of a finite difference method[12,19]. In this article, eq. (1) and eq. (4) are partial differential equations in nature, therefore we have used a tool i.e. MATLAB 2019a to solve these equations. We can solve all partial differential equations along with all supporting equations ordinary differential and algebraic equations with MATLAB.

Results and discussion

A mathematical model has various additional process parameters such as density, specific heat, thermal conductivity, diffusivity, mass flux, of the *Solanum tuberosum* food material and density, velocity, and drying air temperature. These parameters can be solved with the help of literature data by providing suitable input conditions such as initial temperature and moisture content[7,21,22]. The calculated values of the process parameters of *Solanum tuberosum* food material are given in Table 1.

Table 1. Input thermo-physical properties

Parameters	Values
Initial moisture content (M_{s0}) (g/g db)	4.02
Initial temperature (T_{s0}) (°C)	33.2
Density of the food product (g/m^3)	1106.7
Specific heat of the food product (J/kg K)	4237.8
Thermal conductivity of the food product (W/mK)	0.48
Effective diffusivity of the sample (m^2/s)	1.6×10^{-9}
Water activity	0.3114
Mass flux (g moisture/m^{2}s)	5.73×10^{-6}
Drying air temperature (°C)	45
Drying air velocity (m/s)	1.5

3-D profile of temperature and moisture distributions:

After solving the partial differential equations of variables, we got some useful results in three-dimensional forms. In these plots, x label represents the sample's thickness, y label represents the drying time, and the z label represents the moisture and temperature distribution. Fig. 3 and Fig. 4 are the MATLAB originated surface plots and represent the output variables such as moisture content and temperature of the *Solanum tuberosum* food material with drying time

and sample thickness. These plots are helpful to analyze the uniformity of temperature and moisture distribution in food material. The increment in the temperature of a food material depends on the drying time as well as food thickness[24]. Thickness is the directly functioning of thermal diffusivity. If a food material has good thermal diffusivity, heat is easily penetrated inside the food material.

Fig. 3 depicts the analysis of temperature distribution inside the food sample. The initial value of the temperature has been found at 33.2°C. After placing the food sample inside the drying chamber of the indirect type of solar dryer, the temperature of the food sample increases slightly with drying time and attains a temperature of more than 50°C. The experimental study has been conducted for 6 h from 11:00 AM to 15:00.

Fig. 3 also represents the variation in temperature with respect to thickness. The highest temperature of the food material has been observed at the surface of the food material because heat is penetrated from the surface to the core of the food material during solar drying. Therefore, maximum temperature has been sustained at the surface of the food material.
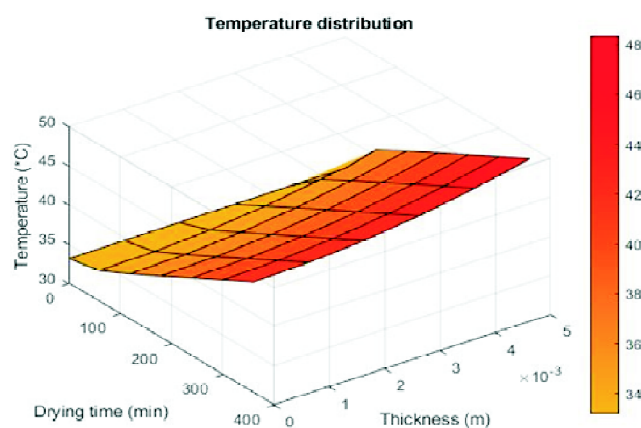

Fig. 3. Temperature distribution across the thickness of the food sample.

Similarly, Fig. 4 depicts the moisture distribution in *Solanum tuberosum* food material along with drying time and sample thickness. Firstly, moisture is transported from the core of the sample to the surface by the diffusion process and then vaporized from the surface continuously until it

reaches its equilibrium state[15]. The moisture evaporation process depends on the increment in the temperature as well as drying time. From Fig. 4, it is also clear that the initial value of the moisture content was 4.02 g/g db. We have assumed that the moisture is uniformly distributed inside the food sample.
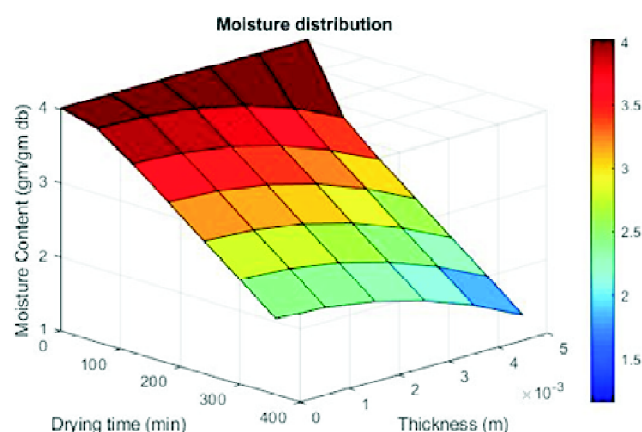

Fig. 4. Moisture distribution across the thickness of the food sample.

As soon as the solar drying process may take place, moisture is evaporated from the surface of food material due to high-temperature conditions and after some time weight loss in food, the material has been observed. After running the six hours of experimental work, the moisture content has attained the equilibrium moisture content condition (no weight loss) up to 0.98 to 1.12 g/g db.

Validation of the mathematical model

To predict the behaviour and check the consistency of the developed mathematical model of potatofood material, experimental work has been done to calculate the temperature and moisture content of food material under the action of an indirect type solar dryer.

Fig. 5 and Fig. 6 represent the validation curves of temperature and moisture content respectively for the model and experimental case at 5 mm thickness of the *Solanum tuberosum* food material. Both figures show that the model data have been found in good agreement with experimental data having a 5% error.
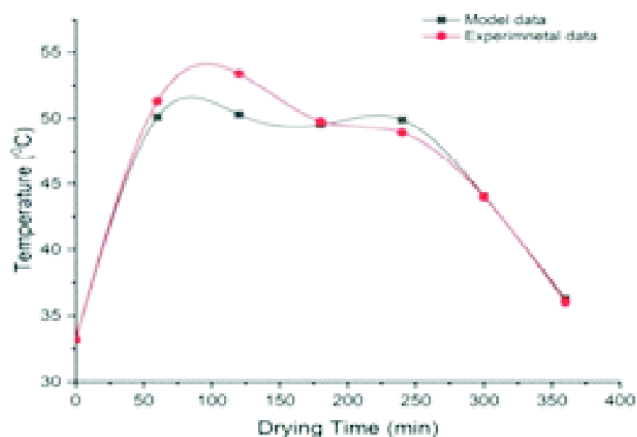
Fig. 5. Variation of temperature with drying time during the validation of model data.
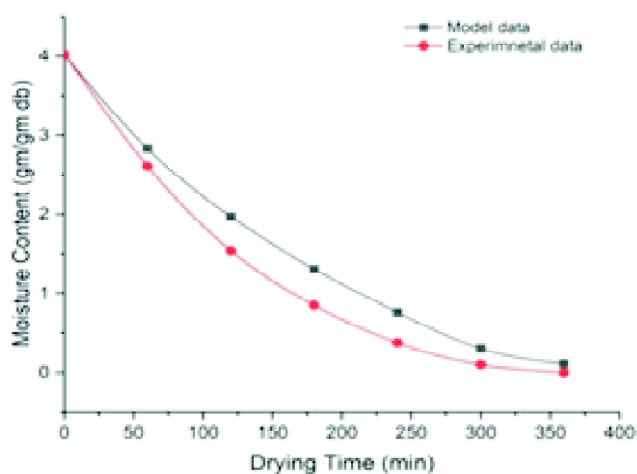

Fig. 6. Variation of moisture content with drying time during the validation of model data.

Conclusion

An experimental and modeling analysis of solar-dried *Solanum tuberosum* have been done with the following conclusions:

(i) Food samples have maintained uniform temperature throughout with a maximum temperature of about 53°C at 13:00 and no hot spot condition takes place.

(ii) The dried *Solanum tuberosum* has attained equilibrium moisture content value 0.98–1.12 g/g.

(iii) The model data have been found in good agreement with experimental data having a 5% error.

(iv) This mathematical model can be used for some other food materials such as carrot, yam, etc.

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