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Power system harmonics estimation using adaptive Kalman filter and its nonlinear variants

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This paper aims to investigate the efficacy of adaptive Kalman filters for power system harmonics estimation. Significant increment of non-linear loads is responsible for the presence of harmonics in power signals which deteriorates the power quality. Towards the improvement of power quality, estimation of the harmonic components is an essential task which has been proposed to be carried out by adaptive Kalman filter and its nonlinear variants. The paper investigates the suitability of adaptation algorithms for harmonics estimation and recommends an appropriate choice of adaptation algorithm. In addition to this, this paper presents a scheme of joint estimation of fundamental frequency along with the harmonic parameters using the nonlinear variants of adaptive Kalman filter. From the relative performance comparison of adaptive nonlinear filters during harmonics estimation adaptive Cubature Quadrature Kalman filter is recommended for power system harmonics estimation for its performance accuracy, numerical stability and reasonable computation cost.

Keywords: Harmonics estimation, cubature filter, cubature quadrature filter, maximum likelihood estimation, Q adaptation.

Introduction

In electrical power systems, power signals get perturbed from pure sinusoidal waveform due to presence of harmonics. Reason behind the existence of such harmonics rich signals is mainly because of the increasing demand of nonlinear loads comprises of power electronics based devices, high power industrial loads, etc. which results in deterioration of power quality[1]. Therefore, it is indeed an essential task to estimate harmonics with accuracy to take corrective actions for power quality improvement. There are several non-parametric methods of harmonic estimation which includes methods based on Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), Least Mean Square (LMS), Recursive Least Square (RLS) and other Recursive algorithms[2]. Kalman filter (KF), reportedly, is a simple and strong candidate for estimation of harmonic parameters of a power signal corrupted with measurement noise and is given preference as it is free from the shortcomings of the other methods[3]. Therefore, the present workers have focused on Kalman filter based harmonics estimation.

However, Kalman filter has a restricted application when non-linearity is introduced in the measurement equation. In such situations the non-linear estimation is carried out by nonlinear variants of Kalman filters out of which Extended Kalman Filters (EKF) is the most popular. In the paper[4] Robust Extended Kalman Filters (REKF) is presented for tracking time-varying harmonic components. But as the degree of non-linearity in the signal increases, the performance of EKF deteriorates due to linearization of significant nonlinearities. As an alternative, UKF has been used in[5], where the harmonic estimation in microgrid is done and UKF outperforms EKF. Still, for high dimension non-linear signal, the accuracy of UKF deteriorates. CKF is, therefore, introduced in[6] based on spherical cubature rule, as an alternative to UKF which leads to nominal computational effort and linearization problem is also taken care of, as CKF is based on non-linear model. The CKF is free from tuning parameters like UKF and has comparable estimation accuracy of UKF.

Several other filters such as Local Ensemble Transform based Kalman Filter (LET-KF) is used in[7] which compared to Ensemble Kalman Filter (En-KF) reported in[8], revealing that LET-KF outperforms En-KF in terms of accuracy and computational efficiency.

In a linear system, the best estimation for a non-adaptive filter is possible only when the measurement and process noise covariances i.e. Q and R are known *a priori*. In practice an arbitrary choice of noise covariance due to the lack of knowledge leads to the divergence of the estimates. For satisfactory performance of Kalman filter proper tuning of Kalman filter is essential. Improper tuning of noise covariances severely degrades the performance of the filter and may cause divergence. Therefore, large number of cases needs to be undertaken for offline tuning of KF. Adaptive Kalman filters can avoid this by online tuning/adaptation of the noise covariance. Early works on KF[9–11] report on auto tuning of noise covariance by online adaptation using the parameter estimation methods, viz. Maximum Likelihood Estimation (MLE), Maximum a Posterior (MAP) respectively. The performance Kalman filter incorporated with the adaptation algorithms for harmonics estimation has been explored in this paper. For static harmonics estimation (where amplitude of harmonics remains constant) usually the system dynamics is hardly affected by system noise and therefore the noise covariance should be of lower value. If the relative difference between the true noise covariance and the noise covariance initialized for the filter is high then the estimation accuracy for non-adaptive KF degrades significantly. AKF can adapt online the inaccurate initial choice of noise covariance and ensure satisfactory estimation result. During the dynamic harmonics estimation where the amplitude of harmonics are time varying the system dynamics is modeled as random walk model with time varying noise covariance. In such situations the use of AKF is highly recommended.

When along with harmonics the fundamental frequency needs to be estimated the measurement equation becomes nonlinear and AKF cannot be employed. For such joint estimation problem adaptive Extended Kalman filter (AEKF) and its successors may be employed. In this work along with AEKF, Adaptive Cubature Kalman Filter (ACKF) and Adaptive Cubature Quadrature Kalman Filter (ACQKF) have been employed and their relative performance has been carried out.

A few works have been reported in literature where adaptive Kalman filters or their nonlinear variants are employed in power system harmonics estimation. A self-tuning Kalman filter algorithm is applied for harmonic estimation in[12] where the harmonic parameters are time varying. The adaptation was performed on the basis of an intuitive adaptation algorithm. In[13] the value of process noise covariance is switched between different values on the basis of a hypothesis framed on t-statistics.

Hybrid genetic algorithm and adaptive particle Swarm optimization based Unscented Kalman Filter (UKF) is developed in[14] to estimate the power system harmonic components. The hybrid Genetic Algorithm and Adaptive Particle Swarm Optimization algorithm is used to estimate the process and measurement noise covariance matrices by minimizing the Root Mean Square Error (RMSE) of the UKF. Here, the approach of standard parameter estimation methods has not been explored.

In this paper a comparative study of three different versions of adaptive Kalman filter has been presented along with its nonlinear version. The contributions of this work are listed below:

Innovation based Q adaptive KF has been employed for harmonic estimation of a power signal perturbed with Gaussian noise where direct adaptation based on MLE method, MAP based Q adaptation and intuitive Q scaling methods of adaptation have been validated.

Performance of these adaptation algorithms has been investigated based on RMS error from Monte Carlo simulation. MLE based direct adaptation of Q is advocated over the other methods of adaptation.

Q adaptation algorithm is also demonstrated for dynamic harmonics estimation where the amplitude of harmonic signal is time varying.

Superiority of Adaptive CKF with MLE based direct Q adaptation is exemplified over its competing algorithm of Q scaling and MAP based CKF and also over non-adaptive CKF during joint estimation of frequency and harmonic parameters with unknown process noise covariance.

New algorithm of Adaptive Cubature Quadrature is also formulated and validated with harmonics estimation problem. This is one of the non trivial contribution of this work. Superiority of the ACQKF is demonstrated over Adaptive CKF during harmonics estimation. The Adaptive Cubature Quadrature Kalman filter has been introduced to demonstrate its superiority amongst its competitors such as Adaptive Cubature filter, Adaptive Extended Kalman filters from the performance comparison carried out in this paper so that this newly formulated adaptive filter may be promoted for real time applications.

Harmonic estimation problem

In harmonic estimation problems, the designer needs to estimate the amplitude and phase of the harmonic components. Along with that, fundamental frequency of the signal may also be needed to be estimated. The nonlinear dynamic systems can be expressed[6] by the following state space equations.

$$\boldsymbol{x_k} = \boldsymbol{f}(\boldsymbol{x_{k-1}}) + \boldsymbol{\mu_k} \tag{1}$$

$$\boldsymbol{y_k} = \boldsymbol{h}(\boldsymbol{x_k}) + \boldsymbol{\upsilon_k} \tag{2}$$

where $\boldsymbol{x_k}$ is the state vector, $\boldsymbol{z_k}$ is the measurement vector, $\boldsymbol{h}(.)$ and $\boldsymbol{f}(.)$ are the nonlinear measurement function and state dynamics respectively. The vectors, $\mu_{\boldsymbol{k}}$, represents Gaussian process noise $\mu_{\boldsymbol{k}} \sim N(\boldsymbol{0},\boldsymbol{Q})$ and the vector $\upsilon_{\boldsymbol{k}}$ represents Gaussian measurement noise $\upsilon_{\boldsymbol{k}} \sim N(\boldsymbol{0},\boldsymbol{R})$.

When frequency estimation is not required the system becomes linear and Kalman filter can be applied in that case for estimation as explained later.

Generally, a power signal containing harmonics and noise can be represented[7] by

$$y_k = \sum_{n=1}^{N} A_n \sin(n\omega kT_s + \phi_n) + A_{dc}e^{-\alpha_{dc}kT_s} + \varepsilon_k \tag{3}$$

where, N represents the number of harmonics, ω is the angular frequency, ϕ_n being the phase of n-th harmonic component. A_{dc} is the DC component amplitude. Sampling time period is given by T_s. k is discrete time step. The exponential term $e^{-\alpha_{dc}kT_s}$ provides the decaying part of DC component. The noise in the measured signal is denoted by ε_k.

For state estimation, we need to simplify the above expression. This is carried out by expanding the exponential term using Taylor's expansion method[7] and ignoring the higher order terms as.

$$A_{dc}e^{-\alpha_{dc}kT_s} = A_{dc} - A_{dc}\alpha_{dc}kT_s \tag{4}$$

Substituting (4) in (3) we get

$$y_k = \sum_{n=1}^{N} A_n \sin(n\omega kT_s + \phi_n) + A_{dc} - A_{dc}\alpha_{dc}kT + \varepsilon_k \tag{5}$$

Further expansion of (5) leads to (6)

$$y_k = \sum_{n=1}^{N} A_n \sin(n\omega kT_s)\cos(\phi_n) + \sum_{n=1}^{N} A_n \cos(n\omega kT_s)\sin(\phi_n) + A_{dc} - A_{dc}\alpha_{dc}kT + \varepsilon_k \tag{6}$$

Eq. (6) can be expressed in terms of state space model as given by (7)

$$\boldsymbol{x_k} = \boldsymbol{F_{k-1}}\boldsymbol{x_{k-1}} + \boldsymbol{\mu_k} \tag{7}$$

where the state transition matrix of dimension $(2N + 3)$ is given by (8)

$$\boldsymbol{F_k} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & -kT_s & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

The state vector is selected as

$$x_k = [x_{1k}\ x_{2k}]^T$$
$$x_{1k} = [A_1\cos\phi_1\ A_1\sin\phi_1 \cdots A_N\cos\phi_N\ A_N\sin\phi_N]$$
$$x_{2k} = [A_{dc}\ A_{dc}\phi_{dc}\ \omega] \tag{9}$$

The measurement equation in linear form can be represented as

$$y_k = \boldsymbol{H}_k x_k + \upsilon_k \tag{10}$$

where,

$$\boldsymbol{H}_k = [\sin\omega k\quad \cos\omega k\ \cdots\ \sin\omega k\quad \cos\omega k\quad 1\quad 1]$$

The nonlinear measurement equation becomes linear when the fundamental frequency, ω is known *a priori*. In that case the order of the system state reduces from $(2N + 3)$ to $(2N + 2)$.

The nonlinear measurement equation can be alternatively expressed as

$$\begin{aligned} h(x_k) = {} & x_k(1)\sin(x_k(2N+3)kT_s) + \\ & x_k(2)\cos(x_k(2N+3)kT_s) + \cdots \\ & x_k(2N)\cos(x_k(2N+3)kT_s) + \\ & x_k(2N+1) - kT_s x_k(2N+1) \end{aligned} \tag{11}$$

Therefore, the amplitude, phase, the decaying DC component of state as related with the state vector as

$$A_n = \sqrt{x_k^2(2N) + x_k^2(2N-1)} \quad (12)$$

$$\phi_n = \tan^{-1}(x_k(2N)/x_k(2N-1)) \quad (13)$$

$$A_{dc} = x_k(2N+1) \quad (14)$$

$$\alpha_{dc} = x_k(2N+2)/x_k(2N+1) \quad (15)$$

Algorithm for adaptive filters

Underlying frame work of Kalman filter:

Kalman filter (KF) is a robust method for estimation of harmonic parameters of a power signal corrupted with measurement noise. KF is simple and robust. It's a Recursive Data Processing Algorithm. For the situation when frequency, ω is known, both the system and the measurement equation becomes linear and Kalman filter can be employed for harmonics estimation. At first the state vector $\boldsymbol{x_k}$ is estimated. Thereafter, the harmonics parameters are obtained as given by (12) to (15). The well known Kalman filter algorithm from[16] has been employed as an underlying framework for adaptive Kalman filter when the process/system noise covariance, $\boldsymbol{Q}$, remains unknown.

Underlying framework of EKF:

When the fundamental frequency needs to be estimated to check its variation, the nonlinear filters are necessitated. Extended Kalman filter (EKF) is widely used nonlinear filter for state estimation which is formulated based on linearization. After linearization it follows the same algorithm for Kalman filter. However, at each prediction and correction steps the nonlinear state equation is linearized about predicted estimate and the nonlinear measurement equation is linearized about corrected estimate. For the brevity of the paper algorithm has not been presented and readers are requested to refer[16].

Underlying framework of CKF and CQKF:

Cubature and Cubature Quadrature Kalman filters are sigma point filters for nonlinear estimation which ensure superior performance over EKF when the degree of nonlinearity in the system or measurement increases. The CKF is a simplified algorithm which can be easily obtained from Cubature Quadrature Kalman filter which is based on Cubature Quadrature rule[15]. Cubature Quadrature rule[15] is based on Spherical Radial rule of numerical approximation of Gaussian integrals. For n-th order integral, the number of quadrature points will be $2nn'$ where n' is the order of radial integration. When $n' = 1$, the Cubature Quadrature rule is reduced to Cubature rule which is same as[17], generated from based on the spherical cubature rule.

Calculation of Cubature Quadrature points and weights: The quadrature points[15], ζ_i are obtained as

$$\zeta_i = \begin{cases} \sqrt{2\lambda_j}\; e_i & \text{for } i = 1, \cdots, n \\ \sqrt{2\lambda_j}\; e_{i-n} & \text{for } i = n+1, \cdots, 2n \end{cases} \quad (16)$$

With weights[15], w_i are obtained as

$$w_i = \frac{1}{2n\Gamma(n/2)} \frac{n'!\Gamma(\alpha + n' + 1)}{\lambda_j\left[\dot{L}_{n'}^{\alpha}(\lambda_j)\right]^2} \quad (17)$$

where λ_i are the roots of n'-*th* order Chebyshev Laguerre polynomial[15], $L_{n'}^{\alpha}$ with $\alpha = n/2 - 1$ given by (18)

$$L_{n'}^{\alpha} = \lambda^{n'} - \frac{n'}{1!}(n' + \alpha)\lambda^{n'-1} + \frac{n'(n'-1)}{2!}(n' + \alpha)(n' + \alpha - 1)\lambda^{n'-2} - \cdots = 0 \quad (18)$$

For more details readers are requested to refer the base paper[15] where from the prediction and correction steps are also taken.

(i) Initialization: Initialize $\hat{\boldsymbol{x}}_{\boldsymbol{o}}$, $\hat{\boldsymbol{P}}_{\boldsymbol{o}}$, $\overline{\boldsymbol{Q}}$, $\overline{\boldsymbol{R}}$

(ii) Prediction step:

Compute Cholesky Factor such that^

$$\hat{\boldsymbol{P}}_{\boldsymbol{k}-1} = \hat{\boldsymbol{S}}_{\boldsymbol{k}-1}\left(\hat{\boldsymbol{S}}_{\boldsymbol{k}-1}\right)^T \quad (19)$$

Choose quadrature points as

$$\hat{\chi}_{\boldsymbol{i}} = \hat{\boldsymbol{S}}_{\boldsymbol{k}-1}\zeta_{\boldsymbol{i}} + \hat{\boldsymbol{x}}_{\boldsymbol{k}-1} \quad (20)$$

$$\text{Compute } \overline{\boldsymbol{x}}_{\boldsymbol{k}} = \sum_{i=1}^{N} \boldsymbol{f}(\hat{\chi}_{\boldsymbol{i}})w_i \quad (21)$$

$$\overline{\boldsymbol{P}}_{\boldsymbol{k}} = \overline{\boldsymbol{Q}} + \sum_{i=1}^{N} \left(\boldsymbol{f}(\hat{\chi}_{\boldsymbol{i}}) - \overline{\boldsymbol{x}}_{\boldsymbol{k}}\right)\left(\boldsymbol{f}(\hat{\chi}_{\boldsymbol{i}}) - \overline{\boldsymbol{x}}_{\boldsymbol{k}}\right)^T w_i \quad (22)$$

$\overline{\boldsymbol{x}}_{\boldsymbol{k}}$ is *predicted* estimate and $\overline{\boldsymbol{P}}_{\boldsymbol{k}}$ is *predicted* error covariance

$\overline{\boldsymbol{Q}}$ denotes the assumed value of process noise covariance. For $\boldsymbol{Q}$ adaptation, $\bar{\boldsymbol{Q}} = \hat{\boldsymbol{Q}}_{k-1}$.

(iii) Correction step:

Compute Cholesky Factor such that

$$\bar{\boldsymbol{P}}_k = \bar{\boldsymbol{S}}_k \left(\bar{\boldsymbol{S}}_k\right)^T \quad (23)$$

Select the points as $\bar{\chi}_i = \bar{\boldsymbol{S}}_k \zeta_i + \bar{\boldsymbol{x}}_k$ (24)

The *predicted* estimate of measurement

$$\boldsymbol{z}_k = \sum_{i=1}^{N} \boldsymbol{g}(\bar{\chi}_i) w_i \quad (25)$$

The following covariance can be computed as

$$\boldsymbol{P}_k^{xz} = \sum_{i=1}^{N} (\bar{\chi}_i - \bar{\boldsymbol{x}}_k)(\boldsymbol{g}(\bar{\chi}_i) - \boldsymbol{z}_k)^T w_i \quad (26)$$

$$\boldsymbol{P}_k^{zz} = \sum_{i=1}^{N} (\boldsymbol{g}(\bar{\chi}_i - \boldsymbol{z}_k)(\boldsymbol{g}(\bar{\chi}_i) - \boldsymbol{z}_k)^T w_i \quad (27)$$

The filter gain $\boldsymbol{K}_k$ is given by

$$\boldsymbol{K}_k = \boldsymbol{P}_k^{xz}\left(\boldsymbol{P}_k^{zz} + \bar{\boldsymbol{R}}\right)^{-1} \quad (28)$$

$$\hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \boldsymbol{K}_k(\boldsymbol{y}_k - \boldsymbol{z}_k) \quad (29)$$

$$\hat{\boldsymbol{P}}_k = \bar{\boldsymbol{P}}_k - \boldsymbol{K}_k\left(\boldsymbol{P}_k^{zz} + \bar{\boldsymbol{R}}\right)\boldsymbol{K}_k^T \quad (30)$$

$\hat{\boldsymbol{x}}_k$ is *a corrected* estimate of state and $\hat{\boldsymbol{P}}_k$ is *a corrected* error covariance.

Adaptation algorithm:

In face unknown process noise covariance, $\boldsymbol{Q}$, the auto tuning of $\boldsymbol{Q}$ is necessitated. Towards this objective three different approaches for adaptation are presented.

(1) Direct $\boldsymbol{Q}$ adaptation based on Maximum Likelihood Estimation[18]:

Compute the innovation sequence as

$$9_k = \boldsymbol{y}_k - \boldsymbol{z}_k \quad (31)$$

The estimated innovation covariance can be computed from a sliding window of epoch length L

$$\hat{C}_{9_k} = \frac{1}{L}\sum_{j=k-L+1}^{k} 9_k(j) 9_k^T(j) \quad (32)$$

Direct adaptation algorithm for $\hat{\boldsymbol{Q}}_k$

$$\hat{\boldsymbol{Q}}_k = \boldsymbol{K}_k \hat{\boldsymbol{C}}_{9k} \boldsymbol{K}_k \quad (33)$$

(2) Scaled Q based on Covariance Matching method[18]:

Adapted $\boldsymbol{Q}$ is obtained as

$$\hat{\boldsymbol{Q}}_k = \lambda_k \hat{\boldsymbol{Q}}_{k-1} \quad (34)$$

$$\text{where, } \lambda_k = \sqrt{\frac{trace\left(\hat{\boldsymbol{C}}_{9_k} - \boldsymbol{R}\right)}{trace\left(\boldsymbol{P}_k^y - \boldsymbol{R}\right)}} \quad (35)$$

(3) $\boldsymbol{Q}$ adaptation based on maximum a posterior method[19]:

Adapted $\boldsymbol{Q}$ is obtained as

$$\hat{Q}_k = \frac{1}{k}\sum_{j=1}^{k}\left\{\left(\hat{x}_j - f(\hat{x}_{j-1})\right)\left(\hat{x}_j - f(\hat{x}_{j-1})\right)^T\right\} \quad (36)$$

Problem statement

A stationary power signal has been given below that contains 1st, 3rd, 5th, 7th and 11th order of harmonics and also a DC component[7,8]. The power signal is corrupted with a measurement noise, μ, of noise covariance $R = 0.0025$.

$$y_k = 1.5\sin(\omega kT_s + 80°) + 0.55\sin(3\omega kT_s + 70°) + 0.2\sin(5\omega kT_s + 45°) + 0.15\sin(7\omega kT_s + 36°) + 0.1\sin(11\omega kT_s + 30°) + 0.5\,e^{-0.5\,kT_s} + \mu_k \quad (37)$$

The objective of this work is to estimate the amplitude, phase, DC component and the fundamental frequency. The process noise covariance to generate the truth model is selected as $\boldsymbol{Q} = 10^{-6} \times \boldsymbol{I}_{12\times12}$. However, for the estimator it is assumed unknown and therefore initialized with an arbitrary choice of $\hat{\boldsymbol{Q}}_o = 10^{-2} \times \boldsymbol{Q}$. The initial value of the true state vector can be obtained using ω = 314 rad/s in (9). The initial choice of state for the filter is chosen as $\hat{\boldsymbol{x}}_o = 0.98 \times \boldsymbol{x}_o$ and $\hat{\boldsymbol{P}}_o = \boldsymbol{I}_{12\times12}$.

Results and discussion

Simulation results for AKF:

The results for the harmonic estimation using Adaptive Kalman Filter (AKF) are given by Figs. 1-4. Figs. 1 to 3 show the true and estimated states. Performance of different Q adaptive KF can be observed here. Fig. 4 presents the Mean

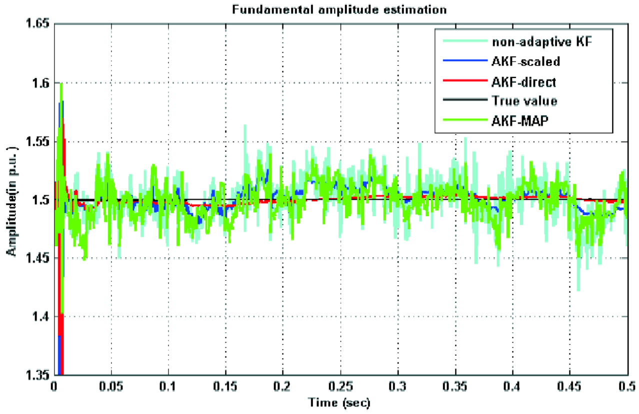

Fig. 1. True and estimated fundamental amplitude for a representative run.

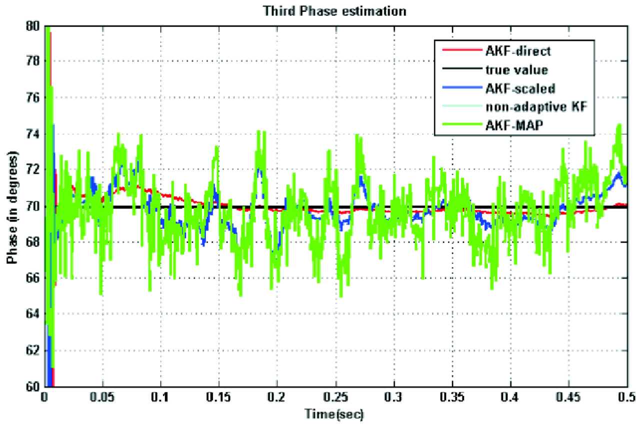

Fig. 2. True and estimated third phase for a representative run.

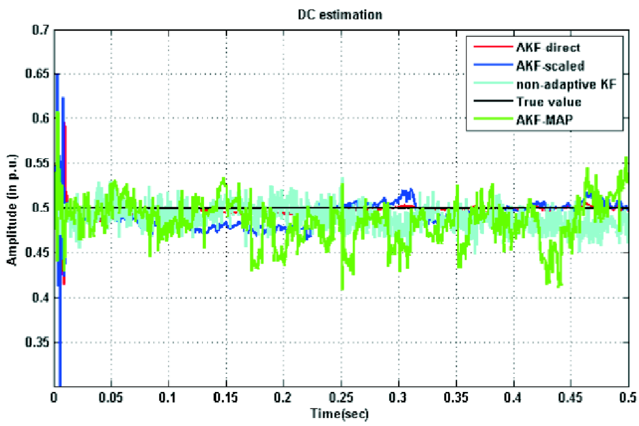

Fig. 3. True and estimated DC amplitude for a representative run.

Square Error of fundamental amplitude from Monte Carlo study with 500 runs. It is observed that for non-adaptive KF

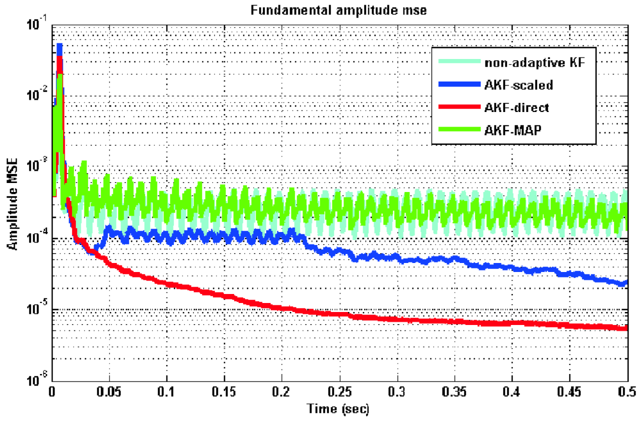

Fig. 4. Fundamental amplitude MSE and error covariance for 500 MC runs.

and also for MAP based method the estimation is not acceptable. Large errors in the estimated values are noted as compared with the true value. For Scaled method, there is considerable improvement in the estimation compared to the other two methods. However, for direct method, a better estimation performance is noted as the error is much lower compared to the other methods. Therefore, the direct method is preferred for adaptation.

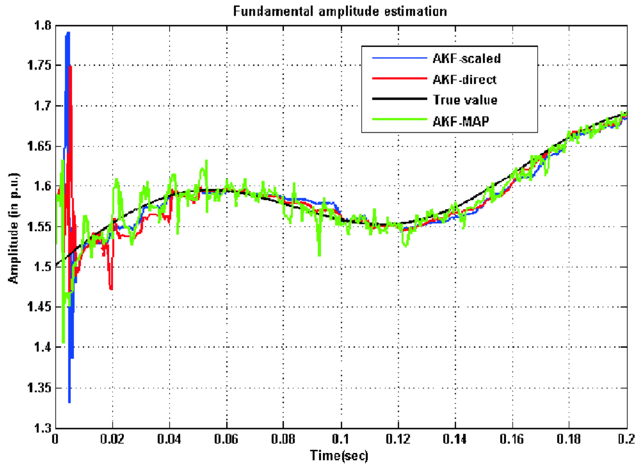

Fig. 5. True and estimated fundamental amplitude for a representative run.

Results for dynamic signal:

The power signals with time varying amplitude are called dynamic signals. AKF is applied to the dynamic signal as given in[7]. The superiority of the MLE based direct adaptation is demonstrated for dynamic signal as well.

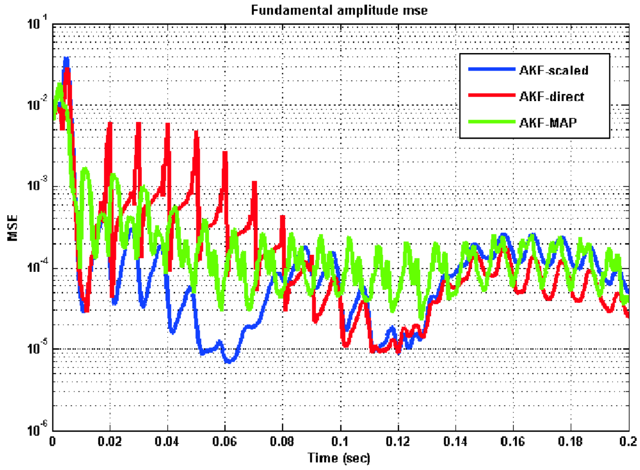

Fig. 6. Fundamental amplitude MSE and error covariance for 500 MC runs.

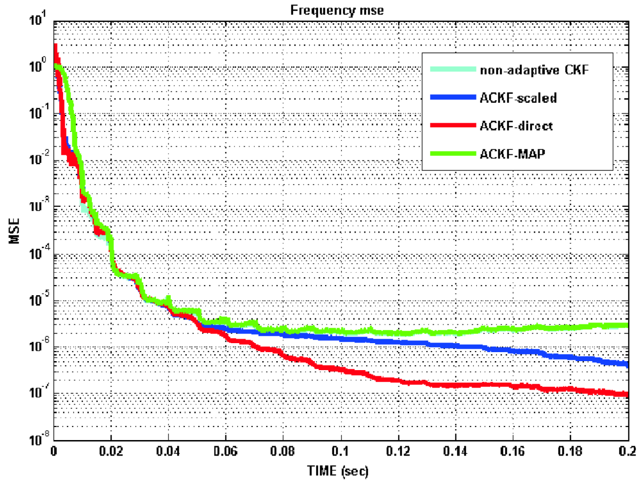

Fig. 8. Frequency MSE for 500 MC runs.

Simulation results for ACKF:

In the above case studies the measurement equation is linear as ω is known. When the frequency needs to be estimated the measurement equation becomes nonlinear. To deal with this problem, Adaptive CKF is applied and a comparative study has been carried out for investigating the best adaptive methods amongst the presented methods.

Parameter values considered for this filter are $\boldsymbol{Q} = 10^{-6} \times \boldsymbol{I}_{13\times13}$ and $R = 0.004^2$. The initial choice remains the same. However, the error covariance taken as $\hat{\boldsymbol{P}}_o = 10^{-3}\times\boldsymbol{I}_{13\times13}$ with $\hat{\boldsymbol{P}}_o$ (11, 11) = 2.

From the previous observations, it is clear that for adaptation methods considered in this paper, MLE based direct adaptation methods gives best results compared with the Scaled and MAP based methods. In this section, a comparative study has been done between two non-linear adaptive filters, i.e. AEKF and ACKF. Direct adaptation method is considered for Q adaptation of both the filters. It is observed from Fig. 9 that ACKF is better than AEKF. Mean Square Error for the estimates is obtained using Monte Carlo simulation with 1000 runs.

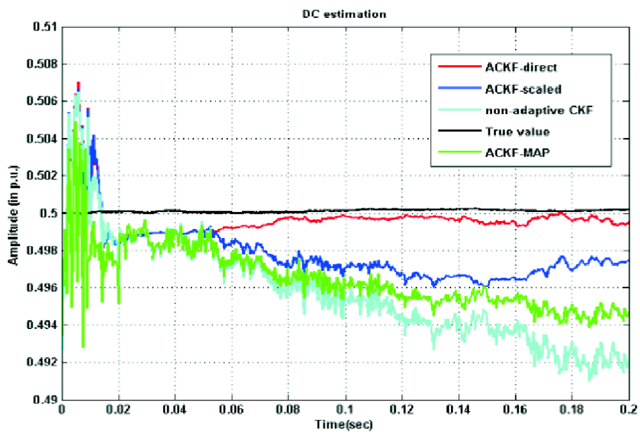

Fig. 7. True and estimated DC amplitude for representative run.

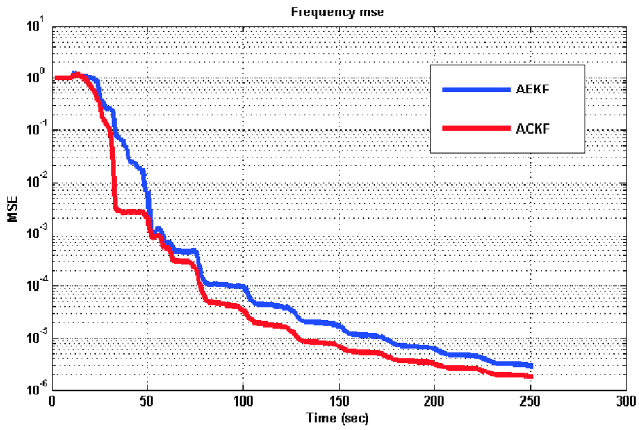

Fig. 9. Frequency MSE for 500 MC runs.

Simulation results for ACKF:

Parameter values considered for this filter are $\boldsymbol{Q} = 10^{-2} \times \boldsymbol{I}_{13\times13}$ and $R = 0.004^2$. The initial error covariance taken as $\hat{\boldsymbol{P}}_o = 10^{-3}\times\boldsymbol{I}_{13\times13}$ where the frequency component is having $\hat{\boldsymbol{P}}_o$ (11, 11) = 2.

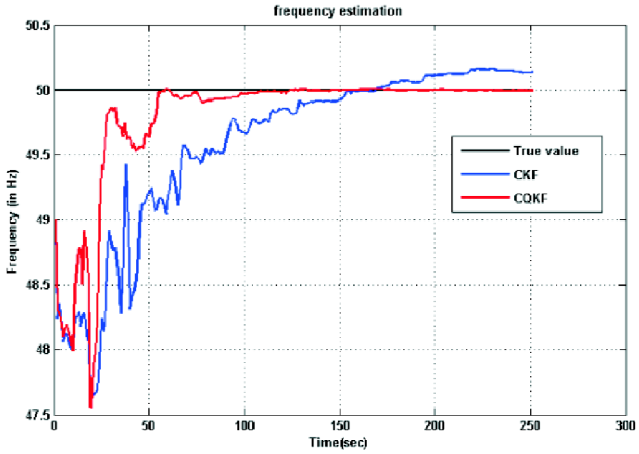

Fig. 10. True and estimated fundamental frequency for a representative run.

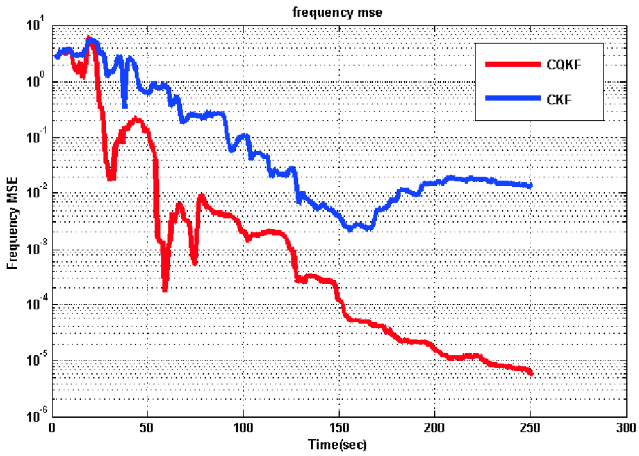

Fig. 11. Frequency MSE for 500 MC runs.

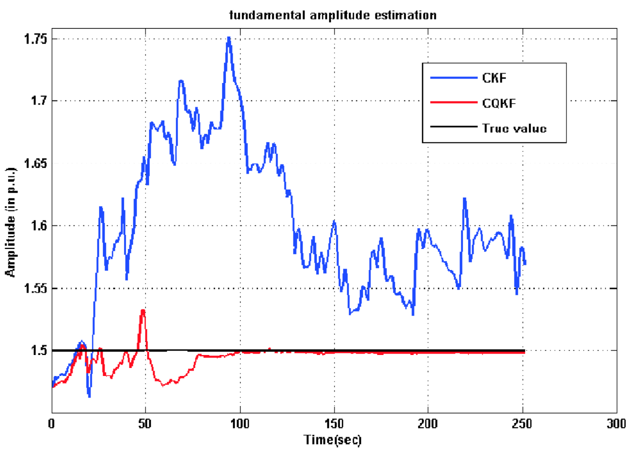

Fig. 12. True and estimated fundamental amplitude for a representative run.

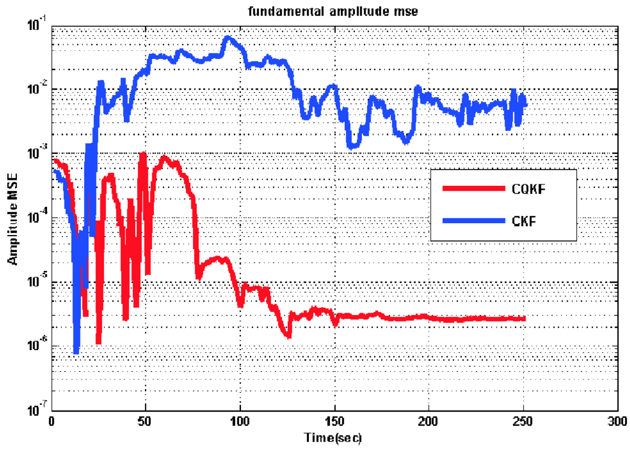

Fig. 13. Fundamental amplitude MSE for 500 MC runs.

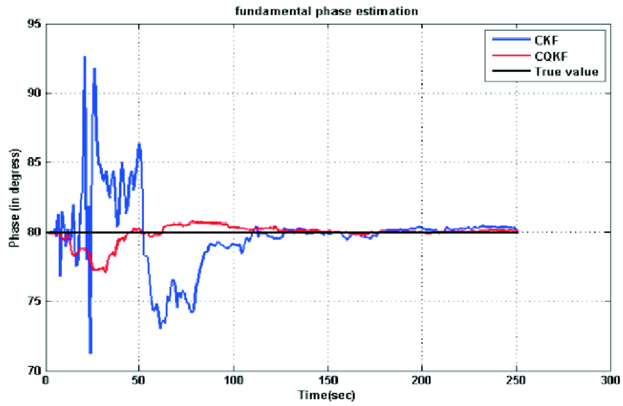

Fig. 14. True and estimated fundamental phase for a representative run.

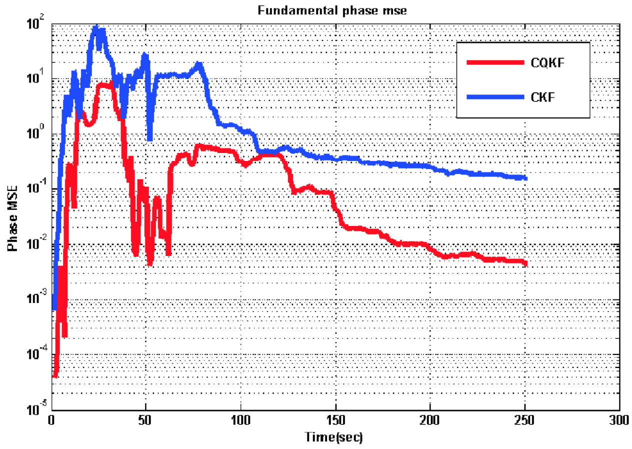

Fig. 14. Fundamental phase MSE for 500 MC runs.

This section represents the comparative estimation results of power system harmonics for ACQKF and ACKF. The frequency estimation is shown in Fig. 10. It is observed that ACQKF gives good estimation accuracy, as the estimated value by it is much closer to the true value as compared to that of ACKF. This is verified by the respective MSE plot as shown in Fig. 11. Note that this estimation results have been obtained for 500 Monte Carlo runs. In the same vein the results are presented for the estimates of fundamental amplitude and phase. We can clearly see that estimated value of ACKF is not following the true value and is also sometimes diverging. The respective MSE plots are shown in Fig. 11, Fig. 13 and Fig. 15. A close look to these MSE plots reveals that mean square error for ACQKF is much less as compared to that of ACKF which indicates its superiority over ACKF.

Concluding discussions

Significant findings from comparative performance analysis of different adaptive Kalman filters along with their nonlinear variants during power system harmonic estimation are enumerated below:

Monte Carlo simulation demonstrates the superiority of Maximum Likelihood Estimation based direct Q adaptation over the MAP based and scaling methods. This is verified for the dynamic estimation of power system harmonics.

For the joint estimation of frequency and the harmonic parameters nonlinear variant of Q adaptive Kalman filters are employed. An adaptive nonlinear filter viz. MLE based Q adaptive Cubature Quadrature Kalman filter has been formulated in this work and its superiority has been demonstrated over the competing algorithm of Q adaptive CKF and EKF.

On the basis of above findings MLE based Q adaptive Kalman filter and Cubature Quadrature Kalman filters are advocated for dynamic estimation of power system harmonics.

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