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Modelling and process design of Moving Bed Bioreactor (MBBR) for wastewater treatment — A Review

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Moving bed bioreactor (MBBR) is an advanced technology for treating both municipal and industrial wastewater. The main operational principle of MBBR is based on bio-film reactor technology under moving state. The important component of this system is bio-carrier on which bio-film grows continuously throughout its entire surface under dynamic condition. Moving bed hybrid bioreactor (MBHBR) is one modification of MBBR, where suspended growth phase is also present due to hydraulic shear from the attached biomass. Mathematical modelling of MBBR process is extremely essential for its rational design in wastewater treatment. Very few research studies have been reported so far pertaining to model development for MBBR and its application in real life situation. The major drawback is that the bio-film under moving state integrated with suspended growth process makes the mathematical model very difficult and complex. Therefore it is absolutely necessary to develop a simplified mathematical model to understand the performance of MBBR/MBHBR more scientifically. The present paper reviews all the existing mathematical models, which have been developed so far on MBBR/MBHBR system. The criteria, applications and limitations of those models are compared in respect of various operational issues. The issue of process design has also been highlighted in the light of different models already developed for MBBR.

Keywords: Moving bed bioreactor, MBBR/MBHBR, mathematical modelling, comparative study, process design.

1. Introduction

In the year of 1985, there are strong political debates among the North Sea countries to significantly reduce about 50% of the nutrient (nitrogen and phosphorus) loads to the North Sea, Norway during the period 1985-1995[1,2]. To meet this challenge there was an urgent need for modification in existing treatment plants as well as installation of new treatment plants[1,3]. As a part of this agenda some initiatives were taken for removal of nutrients, through a National Research and Development programme in Norway to save the North Sea in Europe from nutrient pollution[4]. Consequently two schemes were selected in pilot plant for removal of Nutrients. One is Pre-denitrification for Nitrogen removal and another one is Post-denitrification for removal of Phosphorus[4]. The pilot plant design was based on purely bio-film reactor process with a large tank[5]. The main philosophical thought behind this idea is that bio-film reactor is more compact than traditional ASP, economically suitable, initiates nitrification and denitrification along with BOD removal and requires less space requirements, which is compatible for Norwegian condition[1,3,4]. The main advantages envisaged for this bio-film reactor are (i) treatment requires less area, (ii) treatment performance is sludge independent and (iii) attached bacteria may be utilized in more specific way for various pollutants due to long sludge age. All the existing bio-film reactors including trickling filter and rotating biological contactor have several advantages as well as disadvantages. The main disadvantages of this reactor configuration are low volumetric efficiency, channelling and odour problem, hydraulic instability, blocking of filter bed, mechanical failure, uneven distribution of waste load throughout the entire carrier etc.[4,6,7]. Therefore, to maintain the strict discharge standard of nutrients load in North Sea as notified by Norwegian State Pollution authority, there was also need for necessary improvement in bio-film rector configuration. Such an improved bio-film configuration is Moving bed bioreactor (MBBR), which was first invented in Norway at the end of the 1980's and early 1990s. This new technology is already patented as pure bio-film reactor without any sludge recirculation in the field of wastewater treatment by Prof. Hallvard Ødegaard[7,8].

In past 20 years, considerable research effort has been made on Moving bed bioreactor studying different types of wastewater for removal of BOD/COD and nutrients in laboratory as well as pilot scale study. Current literatures and pilot plant reports reveal that there are more than 600 municipal and industrial treatment plants, based on MBBR process in operation around more than fifty different countries all over the world[6]. Since early 1990s, MBBR system has been extensively used for upgrading the existing treatment plants to improve their efficiency in terms of BOD/COD and nutrients[9]. However, more than 300 MBBR units are still based on empirical design and experiences from previous studies.

There are so many applications of MBBR system in Europe and United States (USA), reported in last two decades. However, in Indian context Moving bed bioreactor (MBBR) has been used mainly for research purpose in laboratory condition. Real life applications of MBBR are still not reported. A performance study was examined by Sahariaha and Chakraborty[10] with high strength wastewater having initial COD concentration 1500 mg/L and thiocyanate concentration (as SCN^-) 800 mg/L respectively using a lab-scale sequential batch moving bed reactor. Experimental combinations were set as anaerobic/anoxic/aerobic mode of operations with a total cycle period (18–36) h. The final effluent characteristics showed removal of (66–80)% COD and (97–99)% SCN^- after one cycle of operation. Recently Hait and Mazumder[11] studied on a shaft-type Moving bed hybrid bioreactor system (MBHBR) treating synthetic carbonaceous wastewater with an average COD concentration ranging between (1000–3500) mg/L in a laboratory scale reactor. The bio-carrier concentration was varied in the range of (10–30) g/L of tyre tube beads. The study revealed that, maximum COD removal of 70.9% was attained under purely suspended condition as compared to almost 90% in case of MBHBR system.

Modelling is an essential tool for appropriate design and operation of any engineering reactor[12]. Unlike the activated sludge plant which has been already modelled using ASM concept developed by International Water Association (IWA)[9,13], MBBR model still depends surface loading rate. It implies that the process design of MBBR is based on pollutant loading rate/unit surface area of bio-carrier and Hydraulic retention time (HRT) for desired effluent quality[9,14]. A very few research studies have been reported towards the development of MBBR model[2,4,9,12,15–25]. The main difficulty in developing MBBR model arises out of application of bio-film model, which is itself very difficult and complex[4,12,19–22]. The key feature of bio-film model is the molecular diffusion of substrate from bulk liquid into bio-film via liquid bio-film interface. The flow rate of substrate through the pathway is denoted as substrate flux (*J*), which depends on the effluent substrate concentration. As a result, the estimation of substrate flux makes the bio-film model more complex than ASM model[4,19–22]. However, some efforts have been made by earlier researchers to find out the solution of classical bio-film model as developed by La Motta[26], Williamson, McCarty[27] and Harremoes[28] to find out the substrate flux (*J*) and other bio-film related output parameters.

The purpose of this present review is mainly to highlight the developments in the modelling of MBBR reactor over last 20 years. It also covers application of different types of MBBR model, their performance, merits, and limitations. It also reviewed the critical aspects of MBBR models and process design issues with a view to predict the effluent substrate concentration, substrate flux (*J*) and the bio-film thickness.

2. Salient features of Moving bed bioreactor

Moving bed bioreactor (MBBR) operates on the basis of non-cloggable aerobic/anaerobic bio-film reactor principle with minimum head loss, and considerable specific surface area[3–5,8,15,29,30]. The key concept behind the development of MBBR reactor is that it combines the working principles of both ASP and bio-filter process simultaneously in a same aeration basin[8,31]. However, original MBBR reactor was patented as pure bio-film reactor[4]. The combination of two technologies results (i) utilization of the entire volume of a reactor without any head loss and clogging problem. It also encourages increase in slow growing microorganism like nitrifier, prompting nitrification in same reactor system[1,32] and (ii) it does not need any sludge recirculation ensuring higher solid retention time[8,33]. These functional attributes are satisfied using moving bio-carriers with attached biomass in aeration basin. A thin bio-film is a layer of biomass, grown on moving carrier particle that can move freely in entire reactor vessel[3,4,31,34,35]. The movement of carrier particle in MBBR system is mainly controlled by aeration system[4,34], whereas the carrier materials are kept in suspension by mechanical mixing[36]. The bio-carrier materials in MBBR reactor are mainly made of light density polyethylene sheets (density slightly less than 1.0 g/cm^3)[3,4,8] to keep the carrier materials easily under floating condition.

MBBR can be considered as a modular treatment system, for combined carbon oxidation, nitrification and denitrification of wastewater in a single reactor without any significant changes in reactor configuration. Furthermore, MBBR system can also be used for treatment of toxic and inhibitory substances with a large variation of influent wastewater. MBBR system combines both ASP and bio-film reactor possesses ensuring advantages over conventional biological treatment units such as (i) lower operation and maintenance cost and smaller footprint, (ii) higher COD removal efficiency, (iii) low growth of filamentous bacteria and improved sludge settling characteristic, (iv) operation at high biomass concentration and (v) higher oxygen transfer efficiency due to continuous movements of bio-carrier[30,31]. The conceptual diagram of a typical MBBR system is shown in Fig. 1.

MBBR reactor has been converted to Hybrid MBBR system (MBHBR), in which some fraction of biomass is kept in suspended form and the remaining portion is in attached state[6]. The inner side of the Moving bed carrier should provide sufficient area for supporting bio-film with minimum loss due to hydraulic shear. The carrier should have specific gravity slightly less than 1, in order to reduce the mechanical energy required for maintaining their floating state. The diffusion coefficient for oxygen transfer rate should also be relatively high on account of its geometrical shape. The maximum specific surface area should be very high for accommodating large amount of attached growth microorganism on the media surface.

3. A brief background of MBBR models

Mathematical model of Moving bed bioreactor is more complex than activated sludge model[19,20]. This is so because of substrate transport from bulk liquid to liquid bio-film interface, substrate diffusion from liquid bio-film interface to bio-film matrix and substrate utilization into the bio-film[27,37]. The introduction of 'substrate diffusion' in bio-film model imparted

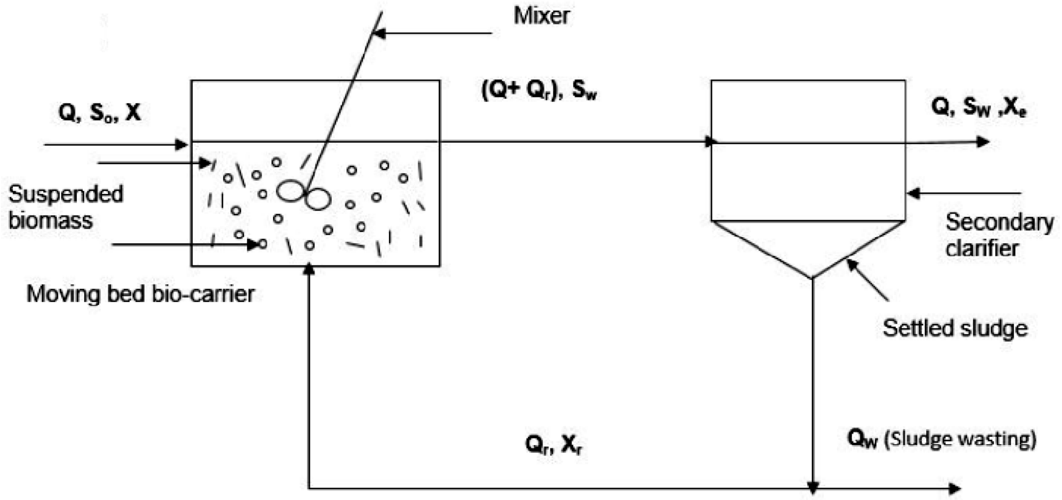

Fig. 1. Conceptual diagram of the MBBR system.

MBBR system is originally based on purely bio-film technology, without any sludge recirculation system and suspended growth microorganism in bulk liquid[2,37]. The bio-carriers are usually made of light polyethylene material of small cylindrical shape on which microorganisms grow up. However, with advancement in wastewater treatment technology,

more complexity in describing the behaviour of bio-film reactor towards the fate of contaminants present in wastewater[2,21,38]. However, various researchers have developed the bio-film models under steady state to study performance of wastewater treatment, the glimpses of which are given below.

3.1. Existing bio-film models

The first steady state bio-film model was developed by La Motta[26] for calculating the mass transfer coefficient in bio-film system under flowing condition[26]. The main objective of that study was to develop the relation between the rate of substrate utilization and fluid flow velocity, to determine the substrate flux and the kinetic regime weather it is metabolism limited or diffusion limited. The model was developed based on certain fundamental considerations such as (i) diffusion of substrate form bulk liquid to liquid bio-film interface, (ii) diffusion of substrate form liquid bio-film interface into bio-film layer and (iii) substrate utilization by bio-film. The fundamental equation was derived from mass balance equations under steady sate as follows.

Rate of substrate utilization = Inflow – Outflow

$$N = \frac{Q}{A}(S_i - S_e) \quad (1)$$

$$N = K_L(S_b - S_s) \quad (2)$$

where, N = rate of substrate consumption/unit area (mol/s/cm^2), Q = flow rate (cm^3/s), S_i = influent substrate concentration (mol/cm^3), S_e = effluent substrate concentration (mol/cm^3), S_b = bulk liquid substrate concentration (mol/cm^3), S_s = substrate concentration at liquid bio-film interface. Under completely mixed flow regime the effluent substrate concentration equals to bulk liquid concentration, i.e. $S_b = S_e$. The value of S_b was calculated form Frank-Kamenetskii model for diffusion equation assuming zero order kinetics (i.e. n = 0) and considering $N = (2 \times D_{ef} \times K_v \times S_s)^{1/2}$, where, D_{ef} = effective diffusivity (cm^2/day), K_v = kinetic coefficient (mol/s/cm^3) and

$$S_s = \frac{Q \times (S_i - S_e)^2}{A \times 2 \times K_v D_{ef}} \quad (3)$$

Hence, it was possible to calculate the value of K_L from eq. (2).

In the same year Willamson and McCarty[27] formulated a steady state mathematical model for substrate utilization by bio-film to understand the kinetic behaviour of different bio-film rectors for wastewater treatment. The conceptual approach was almost same as that followed by La Motta[26]. In this case Monod kinetic expression, simultaneously coupled with Fick's second law of molecular diffusion into the bio-film was used unlike earlier model. Therefore, the substrate utilization by steady state bio-film yields

$$\frac{d^2S}{dz^2} = \frac{k.X_f S}{D_f.(k_S + S)} \quad (4)$$

where, D_f = diffusion coefficient (cm^2/day), k = specific growth rate (day^{-1}), K_s = half saturation coefficient (mg/cc), X_f = active biomass density in bio-film (mg/cc), S = substrate concentration in bio-film at point from the inside face z (mg/cc).

The equation was solved numerically using 4th order Runge Kutta finite difference method under two boundary conditions viz.

(i) at $z = L$, $S = \delta$ and

(ii) at $z = L$, $\frac{ds}{dz} = 0$.

Another important contribution of this model was relationship between electron donor and electron acceptor in bulk liquid with respect to depth of bio-film. Hence, it was possible to check whether the biofilm is metabolism limited or diffusion limited based upon electron donor or electron acceptor.

After pioneering contribution in steady state bio-film by Williamson and McCarty[27], several mechanistic models have been presented by different researchers to describe the substrate utilization and mass transfer phenomena within the bio-film Harris and Hansfort[39] formulated a steady state model quantifying the utilization of biodegradable substrate by bio-film. The purpose of this model was to establish the performance of bio-film reactor under variable organic loadings and to know the effect of substrate i.e. organic carbon and dissolved oxygen (DO) both individually as well as simultaneously on the overall process. The basic assumptions were mostly same as those in previous models. The final expression of this steady state model was presented as

$$\frac{d^2S}{dx^2} = \frac{k.X_f S}{D_f.(K_S + S)}\left(\frac{O}{.(K_o + O)}\right) \quad (5)$$

where, S = substrate concentration at any point within bio-film (mg/cc), X_f = bio-film density (mg/cc), D_f = diffusion coefficient (cm^2/day), z = distance from bio-film fixed surface (cm), K_o = half saturation constant (mg/cc) and O = dissolved oxygen concentration (mg/cc).

Later this model was validated for a vertically mounted bio-film reactor with kinetic coefficients available in relevant literatures. In the same year Herremoes[28], also developed a model, which described the dentrification phenomena in attached growth process. This model was also believed to be an important contribution in bio-film modelling field in earlier 1980s.

The physical models developed by La Motta[26], Williamson and McCanty[27] and Harremoes[28] describing the substrate utilization by attached growth microorganism were further modified by Rittman and McCarty in series of research papers[37,40,41]. The important contribution in the model developed by Rittman and McCarty was that under steady state condition biomass growth rate is exactly equal to biomass decay rate as given below.

$$S = S_0 - A.J.\theta \quad (6)$$

$$L_f = J.Y/(X_f.b_t) \quad (7)$$

where, S = effluent substrate concentration (mg/cc), S_0 = influent substrate concentration (mg/cc), J = substrate flux into the bio-film (mg/cm^2/day), A = specific surface area of attachment surface (cm^{-1}), L_f = bio-film thickness (cm), X_f = active biomass density with the bio-film (mg/cc), Y = yield coefficient (mg/mg).

These equations were derived from basic steady state substrate balance and biomass balance equation in the bio-film. Another important feature of this model was that a minimum substrate concentration in bulk liquid (S_{min}), was considered below which steady state bio-film cannot exist[37,41]. The mathematical expression was proposed as $S_{min} = \frac{K.b_t}{Y.k - b_t}$ [where k = specific substrate utilization rate, K = half saturation constant, b_t = coefficient of total bio-film loss]. The above model was also verified by Chang and Rittmann[42,43] on activated carbon for removal of substrate as well as biomass growth.

Rittmann and Dovantzis[44] also developed a steady state bio-film model, where single substrate or two substrates were rate limiting within the bio-film and it was called dual limitation. The approach of this model was same as that developed by Williamson and McCarty[27,37]. The important contribution of this model concept was that it considered dual limitation with regards to combined removal of soluble BOD and nitrification. It was also suggested that dual limitation should be considered during process design of bio-film reactor as a rational approach. Apart from that, Qi and Morgenorth[45] presented a bio-film model representing simultaneous utilization of electron donor and electron acceptor which was linearly balanced with each other under steady state condition. The analytical solution for this model was not very much complicated compared to earlier classical biofilm models which were to identity the rate limiting substrate concentration in bulk liquid.

Some efforts have also been made towards solution of classical bio-film model Kissel *et al.*[46]; Gujer and Wanner[47]; Sudian and Wang[48]; Annachharte and Khanna[49]; Kim and Suidan[50]; Heath *et al.*[51]; Golla *et al.*[52] to determine various output parameters like steady state bio-film thickness (L_f), effluent substrate concentration (S), substrate flux (J) either by semi empirical algebraic expression or by numerical approach or by normalized loading curves. Recent solution approach of classical bio-film is concerned with pseudo-analytical method using different dimension less expressions[41,53–55]. Mathematical modelling in different programming languages also contributed a lot in recent years towards solution of bio-film problems[56,57].

Strand[58] developed a simplified model describing the combined carbon oxidation and nitrification in aerobic bio-film reactor. Monod's equation and Fick's diffusion equation were used to establish the final model expression. The objective was to predict the COD concentration and ammonium flux with the bio-film using standard kinetic coefficients values from literatures. The analytical solution was made as per Rittmann and McCarty[37] for substrate (COD) flux into bio-film. Apart from that, several steady state bio-film models have been formulated considering the efficacy of attached growth microorganism under toxic environment[59–62]. A summary of bio-film models has been presented in Table 1.

3.2. Existing MBBR models

Mathematical model of Moving bed bioreactor is more complex than activated sludge model[19,20]. The main reason for that is substrate diffusion from bulk liquid to bio-film matrix and simultaneous substrate utilization into the bio-film[41]. Most of the earlier MBBR models are based on Monod's kinetic model assuming either suspended or attached growth or considering both types of growth in sequential order. The fundamental aspects were undertaken on mathematical

Table 1. Summary of existing bio-film model

Sl. no.	Theme of the work	Important findings	Ref.	Critical remarks
1.	Development of simplified bio-film model to find out the expression for external mass transfer coefficient in the bio-film reactor under turbulent condition	The flow velocity and mass transfer coefficient has strong correlation, and it can be determined provided the substrate concentration at liquid-bio-film interface is estimated first	26	The model was not able to find out the effluent substrate concentration, bio-film thickness, and other bio-film related parameters, which was crucial for designing purpose
2.	Development of mathematical model for substrate utilization by attached growth microorganism	Runge-Kutta finite difference technique was applied to solve the second order differential equation of mass balance of substrate in the bio-film and to find out the expression for substrate flux (J) and Bio-film thickness (L_f)	27	The main drawback of said model was that it did not consider the substrate mass balance and biomass balance in terms of Hydraulic retention time (HRT) , Solid retention time (SRT)
3.	Development of steady state bio-film model for single substrate	The important finding is concept of S_{min} in bulk liquid below which steady state bio-film does not exist has been introduced. Another important contribution is under that steady state condition biomass growth is equal to bio-film loss	37	The effective thickness of bio-film could not be analyzed with this model. Moreover no analytical solution was done against second order differential equation of mass balance of substrate in the bio-film
4.	Development of simplified algebraic expression for bio-film model solution	The second order non-linear differential equation was partially solved by using algebraic bio-film expression for single substrate bio-film model. The said model was used to generate the expression for dimensionless substrate concentration at bio-film liquid interface, dimensionless substrate flux, and dimensionless bio-film thickness using nonograph	48	The said model was unable to find out the expression for effective bio-film thickness
5.	Development of bio-film model using normalized loading curves	The model was an approximate solution of second order non-linear differential equation for substrate mass balance to determine the substrate flux (J)	51	The model had major drawback is using normalized loading curves where possible chance of human error
6.	Development of pseudo analytical solution for steady state bio-film	The pseudo-analytical solution is most advance and accurate version of bio-film model solution using various dimensionless variables	53	The effective bio-film thickness cannot be determined by this method. Moreover the solution are very difficult and involves series of complex steps to find out the desirable parameters i.e. effluent substrate concentration, substrate flux and bio-film thickness
7.	Steady state bio-film modelling under dual substrate condition	The model considered, substrate flux limitations depending on either electron donor or electron acceptor concentration in bulk liquid. The said model was solved under steady state condition considering the fact that electron acceptor flux is balanced by electron donor flux. The solution was not very difficult compared to earlier models	45	The model is unable give any clear solution for effective bio-film thickness measure with respect to the electron acceptor and electron donor concentration in the bio-film
8.	Development of mathematical expression for substrate flux	The mathematical expression for substrate flux was found exact, simple, accurate, and easy to solve	72	The major limitation of this model is that it can be used only for known effluent substrate concentration
9.	Development of simplified bio-film model for fixed bed bio-film reactor	The model was very simple and accurate. This second order non-linear differential equation was solved using Runga-kutta 4th order numerical method. The expression of substrate flux were developed as weighted average of different values of substrate flux in the bio-film	73	This model actually gives an expression for solving the effective bio-film thickness, effluent substrate concentration and substrate flux without prior knowledge about the effluent substrate concentration

modelling of MBBR is the competition between both suspended and attached growth microorganism for common electron donor i.e. organics under moving state[9,12,15,17–21]. However, there is a few literatures published earlier in a true sense to develop a hybrid bioreactor model considering both suspended and attached growth simultaneously[64].

Lee[64] has developed a simplified equation for depicting substrate removal in hybrid bio-reactor.

$$S_0 - S - \frac{kSw\theta YaJ\,\dfrac{bs}{bt}}{(K + Sw)\left(\dfrac{1}{\theta c} + bd - \dfrac{YkSw}{K+Sw}\right)} - a.J.\theta = 0 \tag{8}$$

where, J = substrate flux into the bio-film (mg/cm^2/day), S_0 = substrate concentrations in the bulk liquid (mg/cm^3), S_w = effluent substrate concentration (mg/cm^3), k = maximum specific rate of substrate utilization (day^{-1}), θ = empty bed hydraulic detention time (h), θ_c = solid retention time (h), K = half velocity coefficient (mg/cm^3), a = specific surface area of supporting media (cm^{-1}), b_s = biomass loss rate due to shearing from bio-film (day^{-1}), b_t = total biomass loss rate from bio-film (day^{-1}), b_d = biomass decay coefficient (day^{-1}), Y = bacteria yield coefficient. Thereafter other researchers also proposed steady state model for both suspended and attached growth biomass simultaneously[54,65–67].

Hosseiny and Borghei[15] developed a simplified mathematical model for MBBR reactor using Kincannon-Stover kinetic expression and mass balance equation for prediction of COD removal efficiency. Experimental validation was performed using different initial COD concentrations between 225 mg/L to 4370 mg/L at HRT of 24 h. The results of study revealed that diffusion process affected the reaction rate. The correlation between COD removal and substrate utilization rate was COD raised to power 0.513, which indicated both hydraulic and bio-film diffusion were responsible for limiting the reaction rate. However, in MBBR system continuous movements of bio-carrier, minimizes the hydraulic diffusion. Sen and Randall[19,20] developed one mathematical model on MBBR, considering both COD and nitrogen removal and biomass production in the reactor. The substrate utilization kinetics both for suspended as well as attached growth microorganism and substrate diffusion flux within the bio-film were considered in that model, similar to Activated Sludge Model derived by International Association on Water Quality (IAWQ). A large number of kinetic parameters were used in that model making it much complicated and cumbersome. Apart from that, no easy and accurate solution could be obtained from that model because of no explicit relationship between effluent substrate concentration (S) and substrate flux (J) in the bio-film.

Boltz *et al.*[21,22] also proposed similar type of MBBR model considering simultaneous substrate diffusion and utilization within the bio-film. The competition between different group of microorganism, i.e. autotrophs and heterotrophs, for both the electron donor and the electron acceptor was also duly considered. The model assumed similar reaction kinetics for microbial activity within suspended biomass and attached bio-film. The model equations are almost similar to those in ASM2, developed by IAWQ. Plattes *et al.*[17] developed a model for pilot plant MBBR treating municipal wastewater. The MBBR model considered attachment and detachment of bio-film from media surface into the bulk liquid and describing the biomass growth kinetics using the activated sludge model no.1 (ASM1). Later Plattes *et al.*[12] evaluated different bio-kinetic parameters by respirometry method based on above model which was already developed and used for model validation and simulation purpose. However, most of those models are very complicated and difficult for solution. A dynamic mathematical model for Hybrid MBBR reactor for nitrifying bio-filter under tertiary nitrification process was developed and validated using pilot plant data taken from Nether Stowey wastewater treatment plant in UK[14]. His approach was concerned with multispecies bio-film model combining ASM1 for describing bio-film process. Masic *et al.*[68] presented a mathematical model in nitrifying MBBR to determine the oxygen concentration profile within the bio-film matrix. The proposed model considered one-dimensional bio-film process coupled with Monod kinetic expression, which was used to develop the model equations. Later, mass balance and substrate balance equation for conventional (CSTR) model were incorporated in the model equation to improve the performance of model.

A computer modelling was developed by Ferrai *et al.*[9] to determine the kinetic and stoichiometic parameters for both autotropic and hetrotropic microorganism in MBBR treating municipal wastewater of Trento North. The present approach of modelling considered ASM3 and substrate diffusion in the bio-film. The main objective of this model was to estimate the values of different kinetic coefficients more rationally for designing MBBR under real life situation. Similar research work was also performed to determine the values of bio-kinetic coefficients under Hybrid MBBR (HMBBR) system to improve the design practice[23]. The model was based upon Monod kinetic expression and diffusion equation for both suspended and bio-film counterpart applying ASM1 concept.

Some recent study was carried out to quantify the performance of MBBR system by Piculell *et al.*[24] and Revilla *et al.*[25]. Piculell *et al.*[24] studied an effect of bio-film thickness on nitrification potential of MBBR under four different bio-film thicknesses in the range of (200–500) μm. The results showed that bio-film thickness had no effect on nitrification. However, denitrification process retarded with decreasing bio-film thickness i.e. thin bio-film system inhibit the denitrification process. Revilla *et al.*[25] developed a mathematical model treating wastewater containing slowly biodegradable substances for paper and pulp industry. The model considered ASM1 approach and multi-species as well as multi-substrate condition. The mutual interaction of different types of micro-organism and effects of predator microorganism were studied in depth. A brief summary on existing MBBR models is presented in Table 2.

4. Application of different models in MBBR reactor

The Moving bed bioreactor is considered as an advance wastewater treatment option, which has been successfully used for wide range of domestic and industrial wastewater. This system includes benefit of both suspended and attached growth microorganism simultaneously. However, the core principle behind this treatment technology is very complicated and time consuming. The main reason behind this is sub-

Table 2. Summary of existing MBBR models

Sl. no.	Model type	Theme of the work	Important findings	Ref.	Critical remarks
1.	Kinetic Model (Kincan-non-Stover model)	To develop the kinetic model for MBBR treating organic carbon present in wastewater under inhibitory condition	Kincannon-Stover model was found effective for organic carbon removal in MBBR under inhibitory environment	15	No physical model was developed for predicting the effluent substrate under the given condition
2.	Activated sludge model no. 1 (ASM1) developed by International Water Association	Formulation of MBBR model using the activated sludge model i.e. (ASM1) along with classical bio-film model	The proposed model was used by GPSX WWT simulation software for simulation study treating domestic wastewater based on pilot scale MBBR system	17	This model was found very complicated. Apart from that, no process design aspect of MBBR system has been presented in this regard
3.	Activated sludge model no 1 (ASM1), AQUIFAS software	A computer modelling and solution of MBBR was done using AQUIFAS computer programming software	This proposed model can be used to calculate the quantity and surface area of media for optimizing nitrification and denitrifiction	19	The model has given detailed outline of a advance computer modelling of MBBR system. However, the basic requirements of MBBR modelling i.e. simultaneous growth of suspended and attached growth biomass was not considered
4.	Activated sludge model no 1 (ASM1)	A computational Model of MBBR system was formulated	The model solution can determine the total bio-film thickness (L_f)and substrate flux (J) using finite difference method)	20	The proposed model solution was unable to determine the effective biofilm thickness (L_e)
5.	Activated sludge model no 2 (ASM2) developed by International Water Association (IWA)	Development of a mathematical model for MBBR system	The model considered multiple rate expressions based on Monod kinetics for the simultaneous removal of carbonaceous and nitrogenous substances	21	The solution of proposed model was made on the limiting cases (1) zero order kinetics and (2) 1st order kinetics. The present model was unable to solve the effective bio-film thickness
6.	Classical mass balance model in terms of COD and MLSS in CSTR mode of operation	The main focus was to understand the biofilm attachment and detachment mechanism in MBBR system under dynamic condition	Biomass loss from bio-film can be easily calculated by using some standard algebraic expression	74	The model cannot provide any information on bio-film loss due to collision between two moving bed particles

strate diffusion through bio-film and its utilization by bio-film. The bio-film theory is very poorly understood till date, which makes the development of mathematical model for MBBR system and related process design more difficult and cumbersome. Therefore, design of most of these treatment plants, based on MBBR reactor are still on empirical basis and available design information relating to MBBR reactor is very poorly understood. However, some efforts have been made for further development of MBBR model as stated in previous section. This following section highlights on information relating to application of different MBBR models in laboratory scale and field scale.

In early 90's MBBR system was first introduced by Odegaard *et al.*[4] in Norway for enhancing the nutrient removal efficiency of conventional ASP reactor. He performed a series of pilot scale studies, all of which showed a very good removal in terms of COD and nutrients. During this period, some initiatives were taken to design the MBBR system and to find relevant useful values for reactor design purposes from the experiences of pilot scale operation. Consequently, Odegaard *et al.*[4] formulated a very important correlation between COD removal rate and effluent COD concentration as

$$r = a.(S_{COD})^b \tag{9}$$

where r = volumetric removal rate in terms of soluble COD (gram/m^3), a and b are constant and S_{COD} = effluent soluble COD concentration (mg/L).

After plotting a set of 'r' and COD in graph the value for a and b was found as 12.4 and 0.57 respectively. These values were obtained from performance study of a pilot plant treating diary industry wastewater with large variations in COD concentration. Similar type of research work was also carried out by Rusten *et al.*[3] to predict the effluent ammonia concentration for a nitrifying MBBR system treating domestic wastewater in nitrifying MBBR. The prediction model obtained from pilot plant study can be written as

$$r = k.(S_N)^n \tag{10}$$

where r = nitrification rate (g NH_4/m^2.d), S_N = concentration of NH_4-N in reactor (mg/L as N) and k = constant.

In this study, Rusten *et al.*[3] also showed a strong relationship between temperature and activity of nitrifying biomass. He reported that with increase in temperature, nitrifying activity also increased Hem *et al.*[1] also studied nitrification potential in MBBR reactor, which showed both ammonia and oxygen as the rate limiting substrate for nitrification. Empirical equations obtained from pilot plant study depicted that when oxygen was rate-limiting substrate the reaction rate was first order function of oxygen and where NH_4^+-N was rate-limiting substrate the reaction rate was in between half order and first order rate. Two empirical equations were obtained in this regard as $r = 0.28 \times S_{O_2}$ and $r = 1.1 \times S_{NH_4\text{-}N}^{0.7}$.

Hosseiny *et al.*[15] performed a study for describing the kinetic behaviour of MBBR reactor treating synthetic wastewater. Results showed that Kincannon Stover model could be better than Monod model. The constant values were found U_{max} = 8.3402 d^{-1} and K_s = 9.4553 g/L respectively. This research study reveals that using Kincannon Stover model, reactor performance can be satisfactory predicted, provided the constant values are available.

Similar research work was performed by Lin[69] to determine various kinetic coefficients in a MBBR system treating synthetic wastewater. The values obtained from batch study results were U_{max} = 0.696 d^{-1} and K_s = 3.80 mg/L for autotrophic nitrifying microorganism and U_{max} = 0.5621 d^{-1} and K_s = 0.824 mg/L for denitrifying microorganism.

Plattes *et al.*[17] formulated a mechanistic model for MBBR reactor and validated with municipal wastewater. The model was based on ASP model 1 and classical bio-film (1D) model. The model was tested and calibrated in GPX-S software using pilot plant Wastewater Treatment Plant (WWTP) at Hesperange in Luxemburg. The model showed very good agreement with respect to experimental values for prediction of ammonia and nitrate concentration. The measured value and predicted values were 8.6 mg/L and 6.3 mg/L for NH_4-N concentration and 4.6 mg/L and 4.7 mg/L for NO_3^--N concentration respectively.

Trapani *et al.*[70] evaluated some kinetic coefficients in HMBBR system of the municipal wastewater treatment plant at Palermo (Italy) municipal WWTP. The values of relevant kinetic coefficients were found as U_{max} = 0.14 d^{-1} and K_s = 0.15 mg/L for suspended growth bacteria and U_{max} = 0.4 d^{-1} and K_s = 0.85 mg/L for attached growth bacteria.

A very few mathematical models of MBHBR system have been developed in last 20 years of research studies. These models vary from simple kinetic models and/empirical models to complex computer models based on ASM series. The solution procedure of these kinetic and/empirical models are very simple and faster, but do not include fundamental engineering aspects. However, application of these kinetic models varies from location to location and consequently it is very difficult and cumbersome for the site engineers and practitioners to apply these kinetic models and standardize them for different situations and environmental conditions. At the same time, computer based models are not used frequently in engineering design on account of complexity. Hence, a relationship is essential between the simplified kinetic model and the complex computer models that include all fundamental reactions happening in wastewater treatment plants.

5. Development of process design for MBBR

The process design for MBBR system consists of several critical issues like site specific treatment objectives, availability of proper pre-treatment facilities, number of bio-carriers and their physical properties, aeration facilities, including air flow rate, physical dimension of aeration tank, reactor porosity, etc. It is also possible to calculate the optimum HRT (θ) and SRT (θ_c) for a target effluent concentration under a known influent concentration using suitable process design approach. There are several standard process design approaches for municipal and industrial wastewater treatment using ASP or Trickling filter. But very few research efforts have been made so far on the development of suitable process design approach for MBBR system. The process design of most MBBR systems is now purely based on empirical model and different data gathered from past experience. Although, different mathematical models like physical model or statistical model have been developed so for by various researchers on MBBR, these are observed to be incompatible for the process design. It is so because the currently available models are not easy to apply and solution procedure requires a series of complex steps. Therefore it is extremely necessary to develop a suitable design approach for predicting MBBR performance more rationally. Looking into this matter a simple process design procedure is proposed below as per the kinetic relationships for the hybrid bio-reactor developed by Lee[64].

The process design for MBBR has been developed considering the competition between suspended growth and attached biomass for a single electron donor molecule under moving condition as a limiting factor. Simple Monod kinetic expression for substrate utilization and biomass growth simultaneously coupled with Fick's second law of molecular diffusion have been used to derive the basic equations for process design of MBBR. Detachment of biomass from biofilm due to hydraulic shear loss and loss due to collision between moving particle have also been considered in the mass balance equation. The values of different bio-kinetic coefficients i.e. k, K_s, Y and specific bio-film related properties i.e. b_s, b_t, b_d are determined experimentally or taken from standard literatures for determining the various items in process design of MBBR. The important items of process design include – (1) the capacity of aeration tank, (2) aeration facilities, (3) porosity of the reactor, (4) optimum hydraulic retention time and solid retention time. It is observed from the different literatures and practice manuals that optimum concentration of suspended biomass (X) generally lies in between 2500 to 3000 mg/L for good settleability of biomass in the secondary clarifier. The biomass concentration in waste sludge is denoted as X_r, which lies between 10000 and 12000 mg/L, in case of recirculation.

(A) Determination of volume of reactor (V)

The volume of the reactor (V) is calculated as per the following steps:

Step 1: Assume an initial values of MLSS concentration (X), hydraulic retention time (θ) for any particular value of porosity (p), influent concentration (S_0), effluent concentration (S_w), no. of carrier/unit volume (N), air flow rate (Q_a) and specific surface area of bio-carries (a).

Now the value of 'aJ' can be calculated as

$$S_w = S_0 - [a.J.\theta]\left[\left(\frac{K.S.\theta}{k_s+S}\right)\times\left\{\frac{\frac{x_0}{\theta_c}+\left[\frac{b_s.Jt.Y.a}{b_t.p}\right]+\frac{R_{abr}}{p}}{\left(\frac{1}{\theta_C}-\left\{\frac{Y.k.S}{K_S+S}-bd\right\}\right)}\right\}\right].p \quad (11)$$

[Goswami and Mazumder[71]]

in which, J = substrate flux into the bio-film (mg/cm^2/day), S_0 = influent substrate concentration (mg/cm^3), S_w = effluent substrate concentration in bulk liquid (mg/cm^3), k = maximum specific rate of substrate use (day^{-1}), θ = empty bed hydraulic detention time (h), θ_c = mean cell residence time (hrs^{-1}), K = half velocity coefficient (mg/cm^3), a = specific surface area of supporting media (cm^{-1}), b_s = biomass loss rate due to shearing from bio-film (day^{-1}), b_t = total biomass loss rate from bio-film (day^{-1}), b_d = biomass decay coefficient (day^{-1}) and Y = bacteria yield coefficient, R_{abr} = abrasion coefficient (mg/cc/day).

'R_{abr}' can be calculated by using the eq. (12) as given below

$$R_{abr} = 1.294\times(r_1 + r_2)^3\times\left(\frac{\in}{Y}\right)^{0.5}\times n_1\times n_2 \left(\frac{m_1\times m_2}{m_1+m_2}\right)\times\frac{3.56}{(1+0.85K^{0.8})^2} \quad (12)$$

where, r_1 = radius of moving particle 1 including bio-film (cm), r_2 = radius of moving particle 2 including bio-film (cm), m_1 = mass of moving particle 1 including bio-film (g), m_2 = mass of moving particle including bio-film 2 (g), $\in$ = power input per unit mass (m^2/s^3), Y = kinetic viscosity (m^2/s), K = inverse of Reynolds number; Reynolds number can be calculated from the velocity of bio-carrier, which in turns depends on the air flow rate (q)[71], n_1 = numbers of moving particle 1 per unit volume and n_2 = number of moving particle 2 per unit volume. In case of only one type of bio-carrier of uniform size, $r_1 = r_2 = r$ and $n_1 = n_2 = n$.

Step 2: Calculate the value of SRT (θ_c) as fallows:

$$\theta_c = p\times\theta \text{ (without any recirculation)} \quad (13)$$

Step 3: Now MLSS (X) can be calculated as,

$$X = \frac{\frac{x_0}{\theta c}+\left[\frac{b_s.Jt.Y.a}{b_t.\varphi}\right]+\frac{R_{abr}}{\varphi}}{\left[\frac{1}{\theta_C}-\left\{\frac{Y.K.S}{K_S+S}-bd\right\}\right]} \quad (14)$$

Step 4: If calculated value X (X_{cal}) = assumed value of X (X_p), then the volume of the reactor (V, in m^3) can be calculated as,

$$V = Q\times\theta \quad (15)$$

where, Q = wastewater flow rate (m^3/day) and θ = hydraulic retention time (day).

Otherwise, changing the value of p, number of bio-carrier/unit volume (N) and air flow rates new value of ($a\times J$) can be obtained from eq. (11) and revised θ value can be obtained from Step 2. Hence, the revised value of X can be recalculated from eq. (14). This trial process by changing the value of p will be continued till two successive values of X become equal.

Hence finally the volume of the reactor can be calculated from the revised θ value as V (m^3) = Q (m^3/day)$\times\theta$ (day).

The dimension of MBBR i.e. length, width and depth can be obtained from the criteria of uniform fluidization of the bio-carrier with maximum aeration.

(B) Determination of specific surface area (a)

Step 1: Calculate the value J from the following relationship considering the rational design value of X_f.

$$J_{avg} = \sqrt{2\times k\times X_f\times D_f\times(S-S_{min})+K_s\times\ln\left(\frac{K_s+S_{min}}{K_S+S}\right)} \quad (16)$$

Step 2: Calculate the value 'a' from the latest value of (a*J) as calculated earlier.

Step 3: Also estimate the surface area from the number bio-carriers per unit volume (N) and check whether it equals to that from Step 2.

Step 4: If two values of 'a' are not equal then change the value X_f and recalculate 'J' as well as 'a'.

(C) Determination of reactor porosity (p)

The reactor porosity (p) can be calculated from following relationship

$$p = \left(\frac{V-N.V_c}{V}\right) \quad (17)$$

V = bio-reactor empty bed volume (m^3), V_c = bio-carrier volume (m^3) and N = number of particle.

(D) Oxygen requirements

Oxygen is required in the MBBR for biodegradation of organic matter and also for respiration of the biomass in the system. The oxygen requirement in the reactor can be calculated as,

$$O_2 \text{ (in g/day)} = \frac{Q(S_o-S_w)}{f} - 1.42\times Q_w\times X_w \quad (18)$$

where, Q_w = sludge wasting rate (m^3/day)

and 'X_w = biomass concentration in the waste sludge (mg/L).

f = ratio of BOD_5 to ultimate BOD.1.42 = oxygen demand of biomass (g/g). In this case, the removal of only carbonaceous organic matter has been considered.

6. Critical aspect of MBBR model

In early 90s, MBBR was first introduced in Norway to remove excess nutrients mainly (nitrogen and phosphorus) along with organics from wastewater before final discharge into the North Sea. The main reason behind it was to take advantages of bio-film reactor over ASP system for enhancing the nutrient removal efficiency in a smaller footprint. The process was based on bio-film growth on bio-carriers of large specific surface area moving continuously without any significant head loss. Interestingly this reactor was patented originally as pure bio-film reactor. In recent time, suspended growth phase has also been introduced to improve the reactor performance in comparison to traditional MBBR. Therefore, to understand the behaviour of this hybrid MBBR (i.e. HMBBR) having both the suspended and attached phase, a suitable mathematical model is very much required.

There are limited numbers of effort, made so far towards development of the mathematical model for MBBR as well as MBHBR. Apart from that, earlier models were based on empirical equations and experiences collected from several municipal and industrial wastewater treatment unit. Thereafter, some kinetic models like Monod model, Kincannon-Stover model or Haldane model were used to predict the performance of the MBBR system. Recent studies on various model development showed ASM (1 or 2) model (IWA) could be coupled with advanced computer programming prior to derive one or more comprehensive model expression. The main theoretical consideration was same i.e. substrate utilization and biomass growth following Monod's or Haldane's kinetics for both suspended and attached growth. In addition, substrate utilization into the bio-film coupled with Fick's second law of molecular diffusion of substrate into the bio-film from bulk liquid was also considered.

There are many studies conducted towards the development of MBBR model in different forms either empirical models or computer models. However, all this models have specific limitations for application and outcome. In the Moving bed hybrid bioreactor (MBHBR), two different types of microorganisms i.e. suspended growth and attached growth phase act simultaneously in a same aeration tank. Therefore, the model must consider simultaneous utilization of substrate. However, most of the earlier models used substrate utilization by two different types of microorganism in sequential manner. Biomass losses due to hydraulic shear and due to abrasion are two critical incidents under dynamic condition, which contribute also in the steady state suspended biomass concentration, which was not considered in the earlier models. Most of the mathematical models as stated earlier have a major drawback that it has not considered any substrate balance in attached growth emphasizing the parameters like 'specific surface area', 'hydraulic retention time' etc. which are the crucial variable in the design of reactor. Therefore, it is an important issue to address carefully during model development.

Another important issue is development of different biokinetic coefficients for MBBR reactor i.e. K, K_S, Y, k_d and different bio-film related properties; i.e. b_s, b_t and abrasion coefficient (R_{abr}). These kinetic coefficients are very important for model development and validation. Although there are many research works on developing different kinetic coefficients, most of these techniques are very difficult and cumbersome. Therefore, this issue needs to be addressed in future to develop a simplified technique for determining the kinetic coefficients for the treatment of any wastewater in the MBBR/MBHBR.

It is already stated that the existing models are complicated and there is no validation with experimental results. No easy and accurate solution was derived in case of earlier models to calculate the output parameters like effluent substrate concentration (COD or NH_4-N), substrate flux, and effective as well as total bio-film thickness. Some researchers have already proposed some solution of MBBR system using advanced computer programming. However, such solutions are extremely cumbersome and involve a series of steps to determine the output parameters. Moreover after studying extensive literatures it is imperative to state that no model validation result has been reported so far exclusively for MBBR/MBHBR in case of treatment of real wastewater.

7. Future scope of MBBR modelling

Earlier MBBR models developed by different researchers are very complicated and involve tedious calculation which requires a considerable time to find out various output parameters. In this context a mathematical model for MBBR reactor developed by present author can also be used to predict different output parameters like effluent substrate concentration (S_w) , substrate flux (J), total bio-film thickness (L_f) more rationally. It can also be used for process design of MBBR reactor. Different kinetic models for carbon oxidation, nitrification, denitrification, phosphorus uptake and release rate can also be used in the model equation to predict the behaviour of MBBR system towards the nutrients (nitrogen and phosphorus) removal. The variation of different parameters like pH, dissolved oxygen (DO) concentration etc. may also be introduced in respective kinetic expression to study the effects of relevant parameters on effluent substrate concentration. The process design of the MBBR reactor using the simplified model can also be used to optimize the various process parameters like Sludge retention time (SRT), Hydraulic retention time (HRT), Air flow rate (Q_a), and physical volume of the MBBR (V) for a target effluent concentration more precisely. This will facilitate in design and fabrication of MBBR system in a cost-effective manner.

Another important phase regarding the MBBR bio-reactor system is to design the bio-carrier media in which the attached growth microorganism grows up. The main objective is to increase the specific surface area i.e. surface area/ unit volume of bio-carrier to increase the treatment efficiency of MBBR. Apart from that, various combinations of bio-carriers for different types, sizes and shapes can also be employed in MBBR system more precisely to optimize the number of media with respect to biomass concentration and specific surface area in reactor. Apart from that the sensitivity analysis in excel program i.e. variation of different process parameters which affects the overall performance of MBBR system treating wastewater is also an important aspect in case of mathematical model. The optimum ranges of critical parameters and their variation for a particular type of wastewater regime can be also derived from the model equation. Therefore it would be very much easy for the designer to identity the range of critical process parameters for reactor optimization. In this regard, the MBHBR model developed by present authors may be used for precise prediction of the performance of MBBR system under real wastewater environment.

Conclusion

MBBR reactor is an advance treatment option for removal of combined COD and nutrients simultaneously form wastewater. It is also capable to withstand the toxic and inhibitory effects observed in industrial wastewater treatment. The main concept is that both the conventional technologies i.e. activated sludge process and bio-film process are integrated in a same reactor vessel, which ensures benefits of two technologies as a whole. In addition, MBBR system also improves the performance of the conventional bio-film reactor, which is operated without any sludge recirculation and under non-cloggable condition. This system maintains high MLSS concentration and SRT without any problems in sludge separation and thereby improves the sludge settleability exhibiting high SVI value. Simultaneously aerobic, anaerobic, or anoxic condition can also be maintained within a single MBBR, which shows better performance efficiency compared to conventional treatment methods. Most of the earlier MBBR models are found very complex and cumbersome especially on account of bio-film component, solution of which is also difficult. The complexity has also been aggravated due to abrasion loss of attached biomass on account of collision of the bio-carriers. Therefore simplified model development for MBBR system considering concurrent growth of suspended and attached is a very challenging task.

The solution of MBBR model requires for certain assumptions particularly in respect of behaviour of many bio-carrier. There is also no simplified solution practice for the MBBR system available so far. At the same time, neither any analytical validation nor any experimental validation with regard to MBBR model has been reported so far. Most of the existing MBBR models cannot predict the substrate flux (J), bio-film thickness (L_f) and most importantly the effective bio-film thickness (L_e). Therefore, it is urgently required to develop a simplified MBBR model which can be solved easily without any tedious calculation step. Then it would be possible to go for process design of the MBBR for a desired effluent quality provided the respective kinetic coefficients and operating parameters are known.

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